36. Termination of Graph Rewrite Systems (Rept. from ST-II)

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1) EARS
2) AGRS
3) SGRS
4) XGRS

Obligatory Literature

- Uwe Aßmann. Graph rewrite systems for program optimization. ACM Transactions on Programming Languages and Systems (TOPLAS), 22(4):583-637, June 2000.
  - http://portal.acm.org/citation.cfm?id=363914
- Tom Mens. On the Use of Graph Transformations for Model Refactorings. GTTSE 2005, Springer, LNCS 4143
  - http://www.springerlink.com/content/5742246115107431/

36.1 EARS

Problems with GRS

- With graph rewriting, there are some problems:

Termination: The rules of a GRS G are applied in chaotic order to the manipulated graph. When does G terminate for a start graph?

  - Idea: identify a *termination graph* which stops the rewriting when completed

- **Non-convergence (indeterminism)**: when does a GRS deliver a deterministic solution (unique normal form)?
  - Idea: unique normal forms by rule stratification

Additive Termination

- A **termination subgraph** is a subgraph of the manipulated graph, which is step by step completed
- Conditions in the additive case:
  - nodes of termination (sub-)graph are not added (remain unchanged)
  - its edges are only added
- If the termination graph is complete, the system terminates

Example: Collect Subexpressions

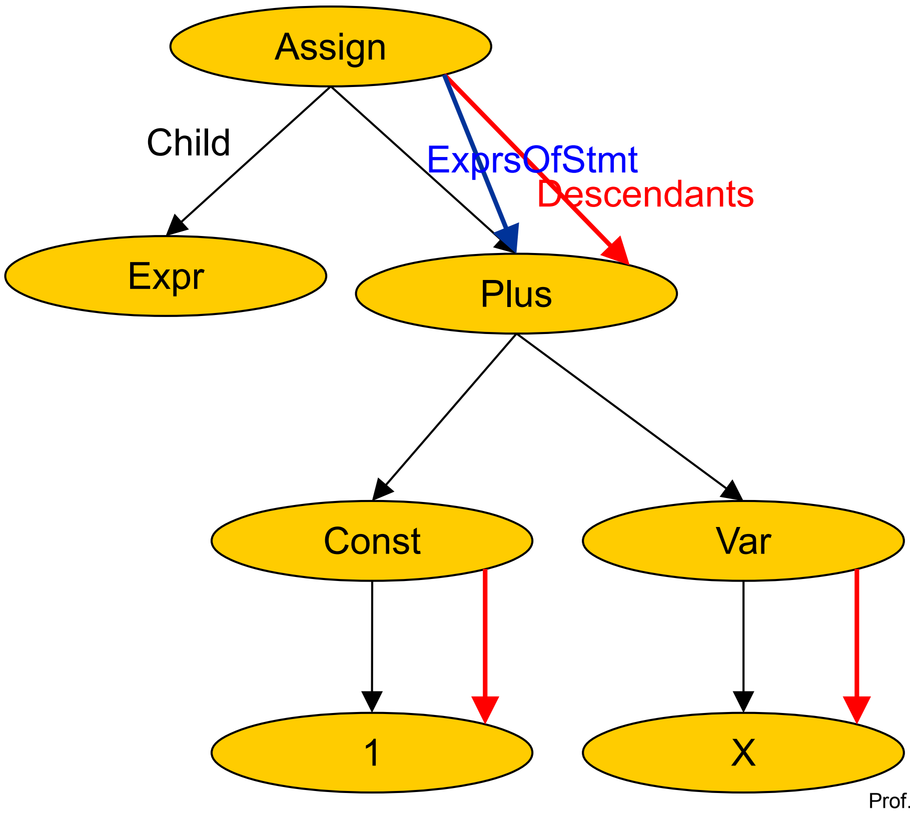
- ''Find all subexpressions which are reachable from a statement''

```
ExprsOfStmt(Stmt,Expr) :- Child(Stmt,Expr).
ExprsOfStmt(Stmt,Expr) :- Child(Stmt,Expr2), Descendant(Expr2,Expr).
// Descendant is transitive closure of Child
Descendant(Expr1,Expr2) :- Child(Expr1,Expr2).
Descendant(Expr1,Expr2) :- Descendant(Expr1,Expr3),
                           Child(Expr3,Expr2).
```

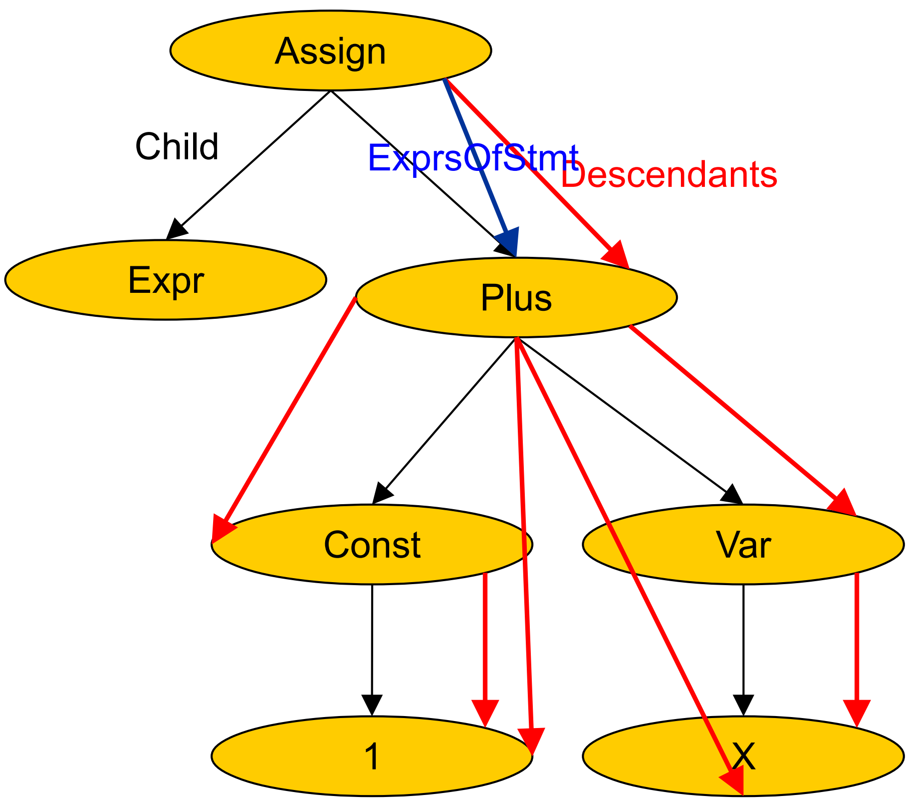
- Features:
  - terminating, strong confluent
  - convergent (unique normal form)
  - recursive
- Why do such graph rewrite systems terminate?

Execution of „Reachable Subexpressions“

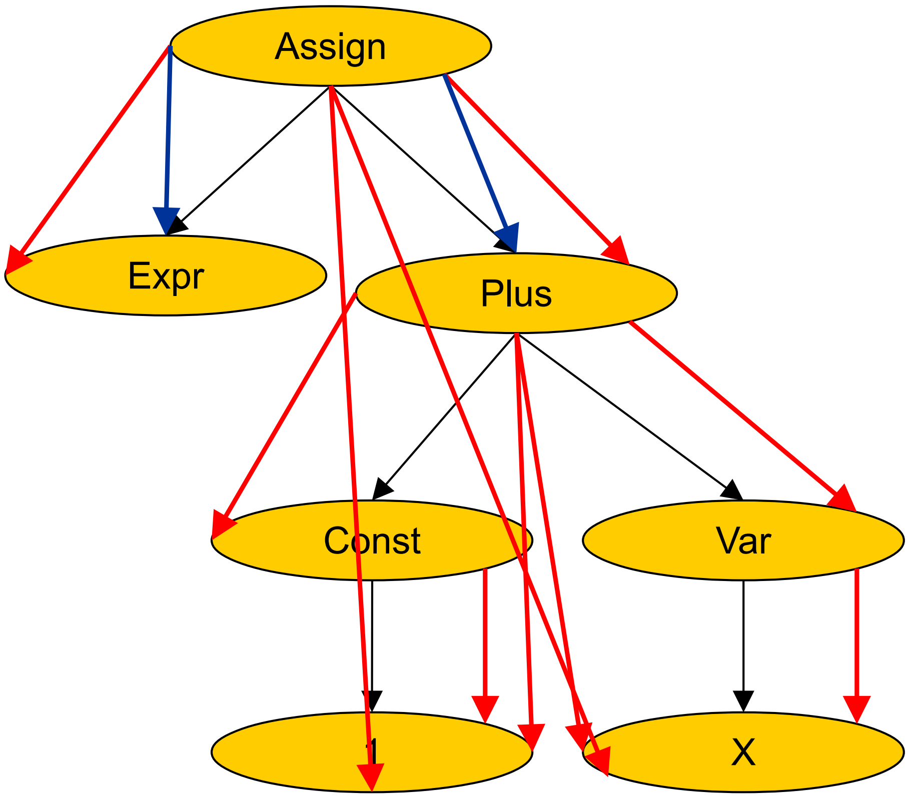
- Answer: ExprsOfStmt and Descendants are termination subgraphs, completed step by step



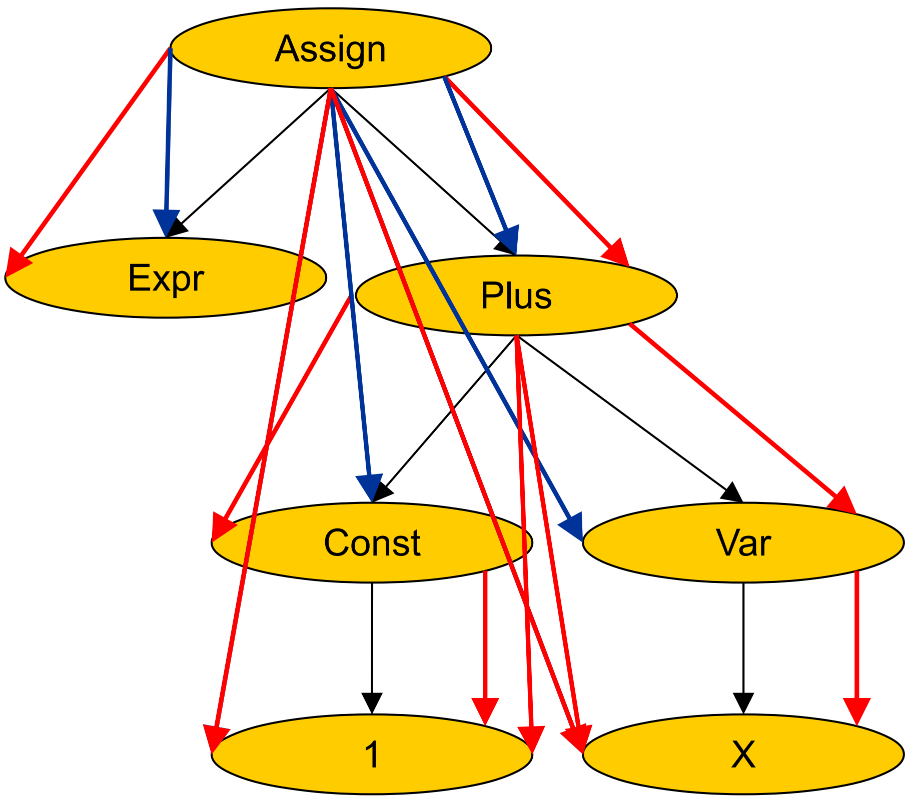
Execution of „Reachable Subexpressions“



Execution of „Reachable Subexpressions“



Execution of „Reachable Subexpressions“



EARS - Simple Edge-Additive GRS

- A subclass of edge-accumulative graph rewrite systems are **EARS (Edge addition rewrite systems)**.
  - They can be used for the construction of graphs
  - For the building up analysis information about a program or a model
  - For abstract interpretation on an abstract domain represented by a graph
- **terminating**: noetherian on the finite lattice of subgraphs of the manipulated graph
  - Added edges form the termination subgraph
- **strongly confluent**: direct derivations can always be interchanged.
- **congruent**: unique normal form (result)
- EARS are equivalent to binary Datalog

Data-flow Analysis with EARS

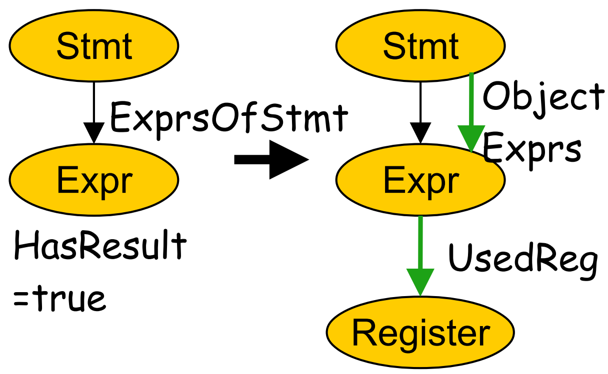
- Every distributive data flow problem (abstract interpretation problem) on finite-height powerset lattices can be represented by an EARS
  - defined/used-data-flow analysis
  - partial redundancies
  - local analysis and preprocessing:
- EARS work for other problems which can be expressed with DATALOG-queries
  - equivalence classes on objects
  - alias analysis
  - program flow analysis

36.2 Additive GRS (AGRS)

Example: Allocation of Register Objects

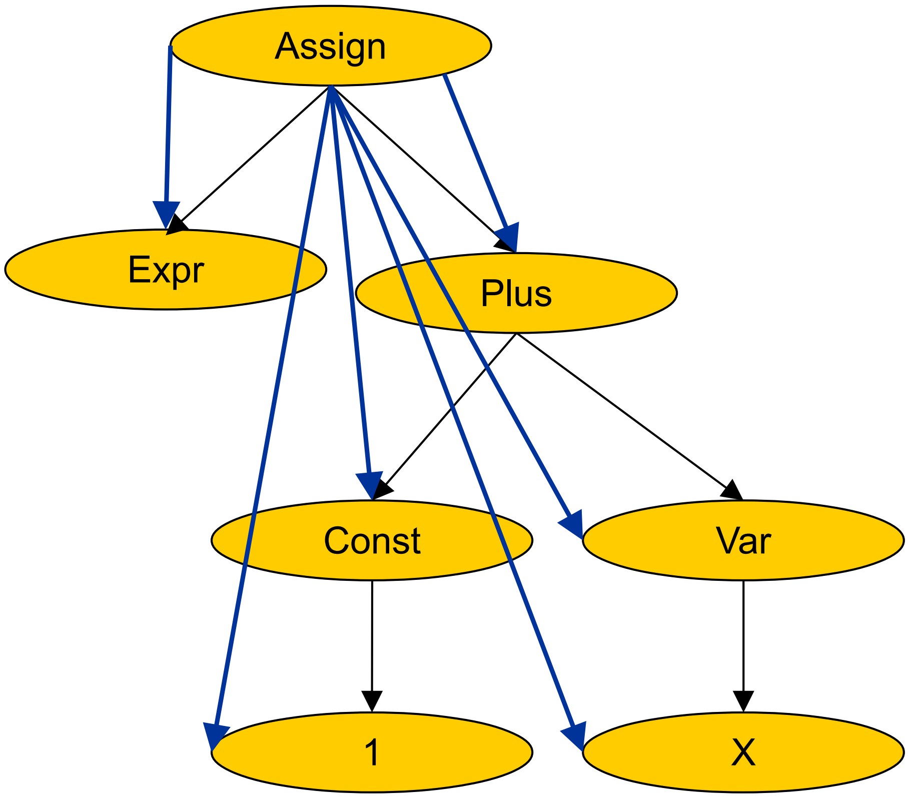
- "Allocate a register object for every subexpression of a statement which has a result and link the expression to the statement"

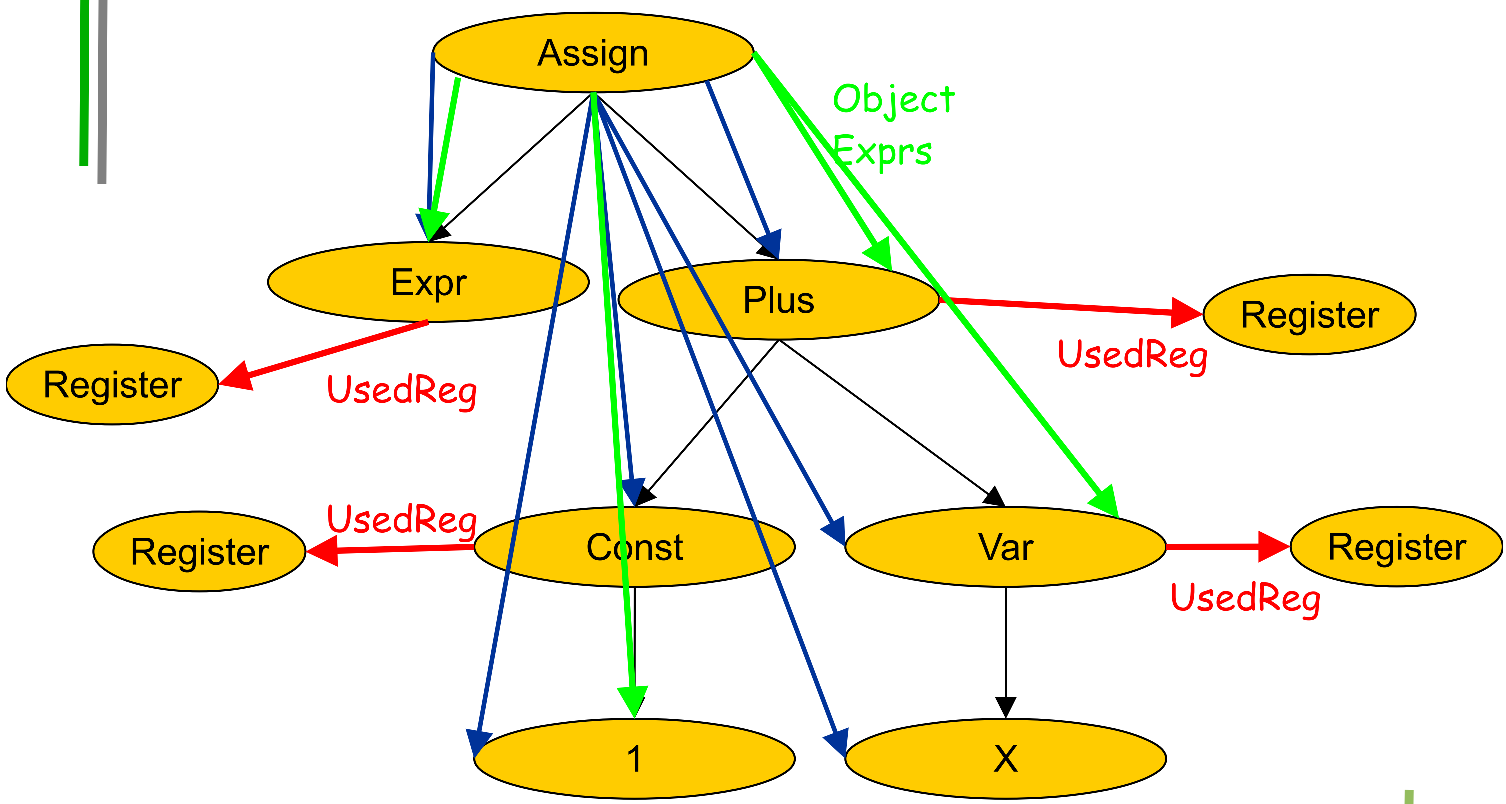
```
if ExprsOfStmt(Stmt,Expr), HasResult(Expr)
then
  ObjectExprs(Stmt,Expr),
  RegisterObject := new Register;
  UsedReg(Expr,RegisterObject)
;
```



- Features: terminating

- ObjectExprs is the termination subgraph





Edge-accumulative Rules and AGRS

- A GRS is called **edge-accumulative (an AGRS)** if
  - all rules are edge-accumulative and
  - no rule adds nodes to the termination-subgraph nodes of another rule.
- Edge-accumulative rules are defined on label sets of nodes and edges in rules
- Criterion statically decidable

The Termination Subgraph of the Examples

Collection of subexpressions:

T = ({Stmt,Expr}, {ExprsOfStmt, Descendant})

Allocation of register objects:

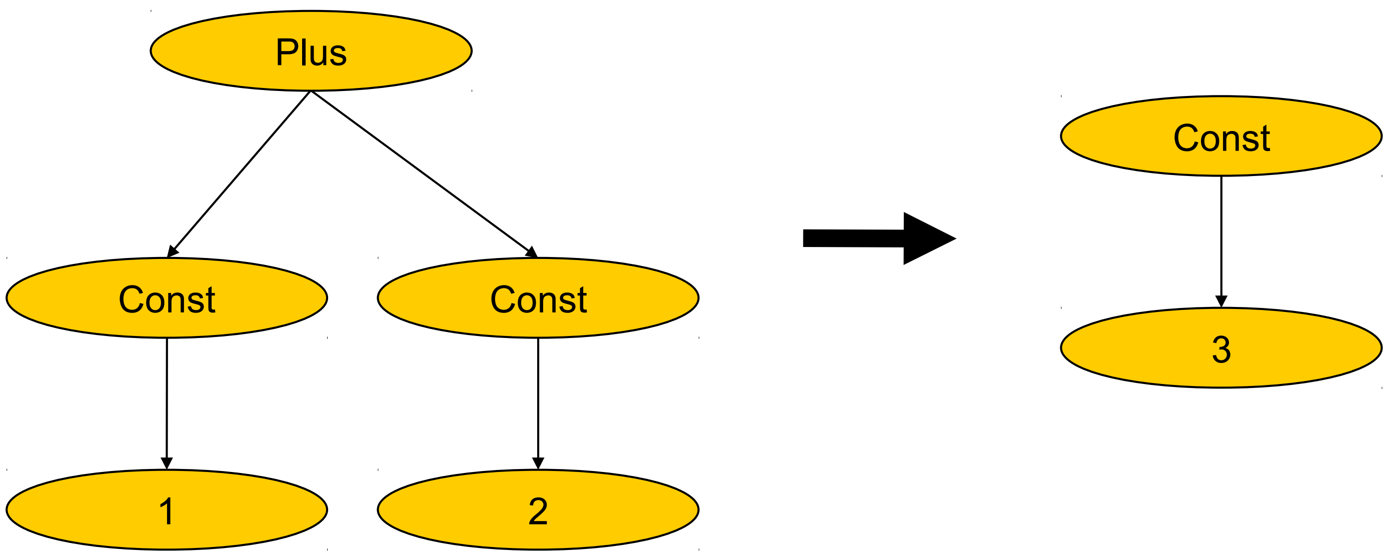
T = ({Proc,Expr}, {ObjectExprs})

36.3 Subtractive GRS (SGRS)

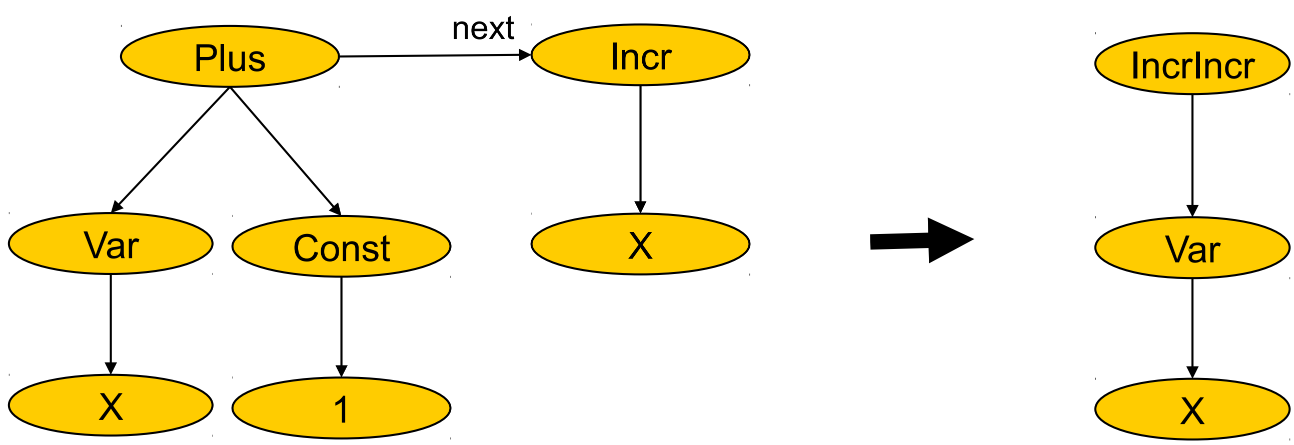
Subtractive Termination

- Conditions in the subtractive case:
  - the nodes of the termination subgraph are not added (remain unchanged)
  - its edges are only deleted
- If the termination subgraph is empty, the system terminates
- Results in:
  - **edge-subtractive GRS (ESGRS)**
  - **subtractive GRS (SGRS)**

Constant Folding as Subtractive GRS



Peephole Optimization as Subtractive XGRS



36.4 Exhaustive GRS (XGRS)

The Nature of Exhaustive Graph Rewriting (XGRS)

AGRS, SGRS make up **XGRS (*eXhaustive* Graph Rewrite Systems)**

All redex parts in the termination-subgraph of the host graph are reduced step by step.

- The termination-subgraph is either *completed* or *consumed*
  - Edge-accumulative systems may create new redex parts in the termination-subgraph, but
    - there will be at most as many of them as the number of edges in the termination-subgraph.
  - Subtractive systems do not create sub-redexes in the termination-subgraph but destroy them.
- XGRS can only be used to specify algorithms which
  - perform a *finite* number of actions depending on the size of the host graph.

The End

- Termination criteria build on a *termination subgraph* that is completed or deleted during the transformation