# 3. Modelling Dynamic Behavior with Petri Nets

Prof. Dr. U. Aßmann Technische Universität Dresden Institut für Software- und Multimediatechnik Gruppe Softwaretechnologie http://st.inf.tu-dresden.de Version 11-0.3, 10/19/11 1) Basics

1) Elementary Nets

- 2) Colored Petri Nets
- 2) Patterns in Petri Nets
- 3) Refactorings
- 4) Composability of Colored Petri Nets
- 5) Parallel Composition with CPN
- 6) Application to modelling



### **Obligatory Readings**

- Balzert 2.17
- Or Ghezzi Chap 5
- or (not enough in Pfleeger):
- W.M.P. van der Aalst and A.H.M. ter Hofstede. Verification of workflow task structures: A petri-net-based approach. Information Systems, 25(1): 43-69, 2000.
- Kurt Jensen, Lars Michael Kristensen and Lisa Wells. Coloured Petri Nets and CPN Tools for Modelling and Validation of Concurrent Systems. Software Tools for Technology Transfer (STTT). Vol. 9, Number 3-4, pp. 213-254, 2007.
- J. B. Jörgensen. Colored Petri Nets in UML-based Software Development – Designing Middleware for Pervasive Healthcare. www.pervasive.dk/publications/files/CPN02.pdf
- Web portal "Petri Net World" http://www.informatik.unihamburg.de/TGI/PetriNets/



### Literature

S

- K. Jensen: Colored Petri Nets. Lecture Slides http://www.daimi.aau.de/~kjensen Many other links and informations, too
  - www.daimi.aau.dk/CPnets the home page of CPN. Contains lots of example specifications. Very recommended
- K. Jensen, Colored Petri Nets. Vol. I-III. Springer, 1992-96. Landmark book series on CPN.
- T. Murata. Petri Nets: properties, analysis, applications. IEEE volume 77, No 4, 1989.
- W. Reisig. Elements of Distributed Algorithms Modelling and Analysis with Petri Nets. Springer. 1998.
- W. Reisig, G. Rozenberg: Lectures on Petri Nets I+II, Lecture Notes in Computer Science, 1491+1492, Springer.
- J. Peterson. Petri Nets. ACM Computing Surveys, Vol 9, No 3, Sept 1977
- http://www.daimi.au.dk/CPnets/intro/example\_indu.html Prof. U. Aßmann, Softwaretechnologie II

### Relationship of PN and other Behavioral Models

- P.D. Bruza, Th. P. van der Weide. The Semantics of Data-Flow Diagrams. Int. Conf. on the Management of Data. 1989
  - http://citeseer.ist.psu.edu/viewdoc/summary?doi=10.1.1.40.9398
- Other courses at TU Dresden:
  - Entwurf und Analyse mit Petri-Netzen
  - Lehrstuhl Alg. u. log. Grundlagen d. Informatik
  - Dr. rer. nat. W. Nauber
  - http://wwwtcs.inf.tu-dresden.de/~nauber/eapn10add.html



### Goals

- Understand untyped and Colored Petri nets (CPN)
- Understand that CPN are a verifiable and automated technology for safety-critical systems

### **The Initial Problem**

• You work for PowerPlant Inc. Your boss comes in and says:

Our government wants a new EPR reactor, similarly, in the way Finland has it. How can we produce a verified control software? We need a good modelling language. Assembler would be too bad...

UML does not work...

How do we produce software for safety-critical systems?



### Interesting Projects with Safety-Critical, Parallel Embedded Software

- Arial
  - The WITAS UAV unmanned autonomously flying helicopter from Linköping http://www.ida.liu.se/~marwz/papers/ICAPS06 System Demo.pdf
- Automotive
  - Prometheus: driving in car queues on the motorway
    - http://www.springerlink.com/content/j06n312r36805683/
- Trains
  - www.railcab.de Autonomous rail cabs
  - www.cargocab.de Autonomous cargo metro
    - http://www.cargocap.de/files/cargocap\_presse/2005/2005\_01\_12%20krus e.pdf
  - http://www.rubin-nuernberg.de/ Autonomous mixed metro

### **Application Areas of Petri Nets**

- Model introduced by C.A. Petri in 1962.
  - Ph.D. Thesis: "Communication with Automata".
  - Over many years developed within GMD (now Fraunhofer, FhG)
  - PNs describe explicitly and graphically: Conflict/non-deterministic choice, concurrency
- Reliable software (quality-aware software)
  - PetriNets can be checked on deadlocks, liveness, fairness, bounded resources
- Safety-critical software that require proofs
  - Control software in embedded systems or power plants
- User interface software
  - Users and system can be modeled as separate components
- Hardware synthesis
  - Software/Hardware co-design

### Application Area I: Behavior Specifications in UML

- Instead of describing the behavior of a class with a statechart, a CPN can be used
- CPN have several advantages:
  - They model parallel systems naturally
  - They are compact and modular, can be reducible
  - They lend themselves to aspect-oriented composition, in particular of parallel protocols
  - They can be used to generate code, also for complete applications
  - UML statecharts, data flow diagrams, and activity diagrams are special instances of CPN
- Informal: for CPN, the following features can be proven
  - Liveness: All parts of the net do never get into a dead lock, i.e., can always proceed
  - Fairness: all parts of the net are equally "loaded" with activity
  - K-boundedness: the data that flows through the net is bound by a threshold



### Application Area II: Contract checking (Protocol Checking) for Components

- Petri Nets describe behavior of components (dynamic semantics)
  - They can be used to check whether components fit to each other
- Problem: General fit of components is undecidable
  - The protocol of a component must be described with a decidable language
  - Due to complexity, context-free or -sensitive protocol languages are required
- Algorithm:
  - Describe the behavior of two components with two CPN
  - Link their ports
  - Check on *liveness* of the unified CPN
  - If the unified net is not live, components will not fit to each other...
- Liveness and fairness are very important criteria in safety-critical systems





### 3.1 Basics of PN

Petri Net Classes

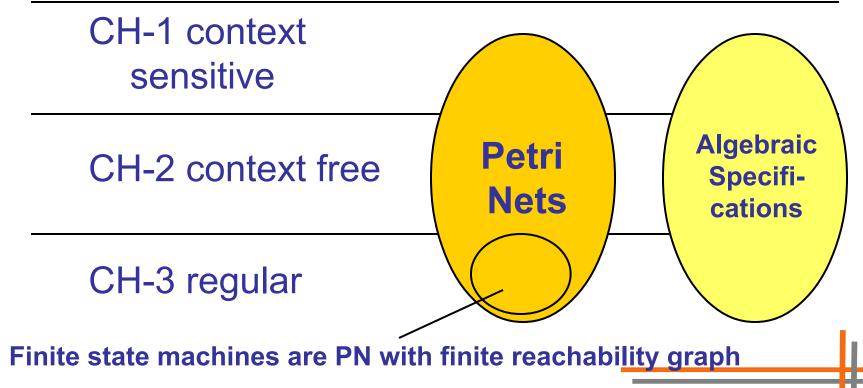
Predicate/Transition Nets: simple tokens, no hierarchy. Place-Transition Nets: multiple tokens High Level Nets: structured tokens, hierarchy There are many other variants, e.g., with timing constraints

### Language Levels

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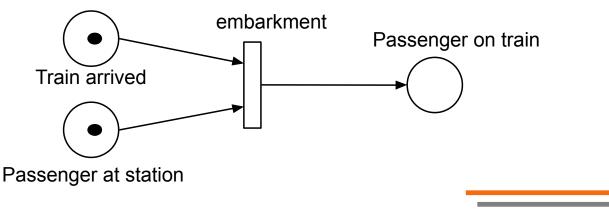
- PN extend finite automata with indeterminism
  - Asynchronous execution model (partial ordering)

### CH-0 computable



### **Elementary Nets: Predicate/Transition Nets**

- A Petri Net (PN) is a directed, bipartite graph over two kinds of nodes, namely places (circles) and transitions (bars or boxes)
- An elementary PN is with boolean tokens, i.e., one token per place (bound of place = 1)
  - aka basic, predicate/transition nets (PTN), condition/Event nets
  - The presence of a token in a place means that the condition or predicate is true
  - The *firing* of a transition means that from the input predicates the output predicates are concluded
  - Thus elementary PN can model simple forms of logic

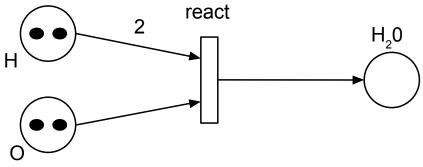




### **Integer Place/Transitions-Nets**

- An integer PN is a directed, weighted, bipartite graph over places and transitions with integer tokens, i.e., places may contain several tokens, and a capacity (bound = k)
  - *k* tokens in a place indicate that *k* data items are available
  - *M(p)* is the number of tokens in place *p*
- A *marking* assigns to each place a nonnegative integer
  - A marking is denoted by *M*, an m-vector where m is the number of places.
  - A PN has a *initial marking*, M<sub>0</sub>.
- Arcs have cardinalities (weights) to show how many tokens they

transfer



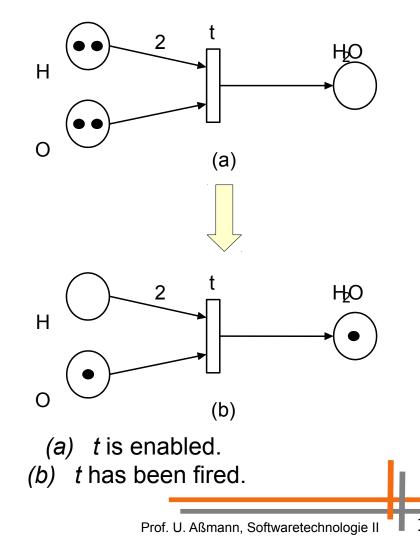


Here: initial marking M<sub>0</sub>(2,2,0) Prof. U. Aßmann, Softwaretechnologie II

### **Formal Transition Enabling and Firing**

In a PN a state is changed according to the following *transitions firing rule*:

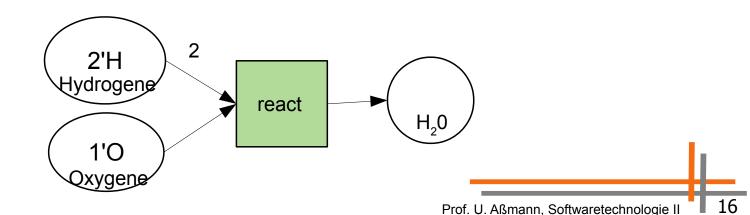
- A transition t is *enabled* if
  - each input place p of t is marked with at least w(p,t) tokens, where w(p,t) is the weight of the arc from p to t
  - The output place can be filled
- An enabled transition may or may not fire.
- A firing of an enabled transition removes w(p,t) tokens from each input place p to t, and adds w(t,p) tokens to each output place p of t, where w(t,p) is the weight of the arc from t to p.





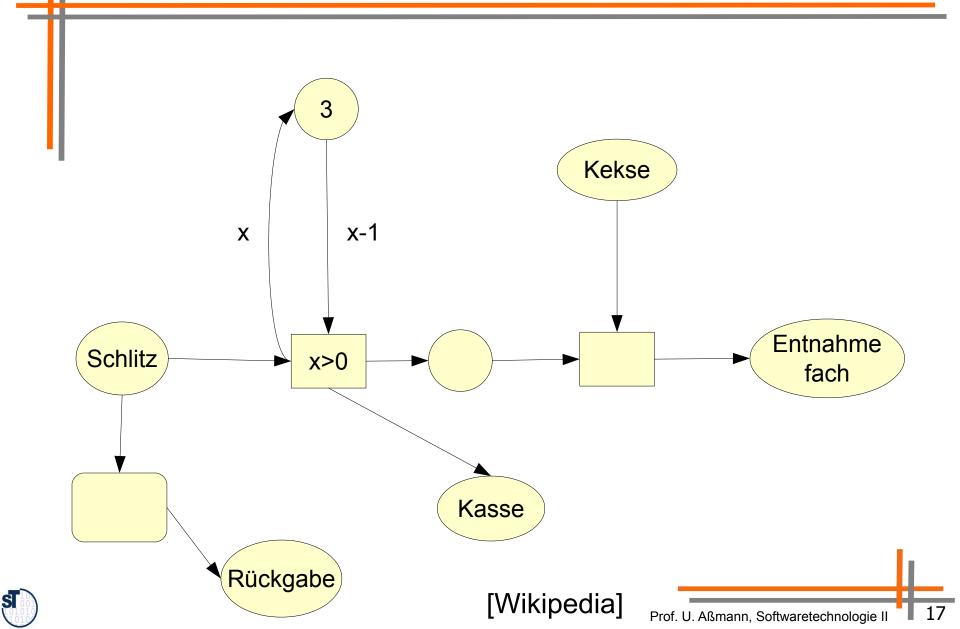
### **High-Level Nets**

- A high-level PN (colored PN) allows for typed places and arcs
  - For types, any DDL can be used (e.g., UML-CD)
- High-level nets are modular
  - Places and transitions can be refined
  - A Colored Petri Net is a reducible graph
- The upper layers of a reducible CPN are called *channel agency* nets
  - Places are interpreted as channels between components





### **Cookie Automaton with Counter**



### 3.1.1 Elementary Nets (Predicate/Transition Nets)



### Meaning of Places and Transitions in Elementary Nets

- Predicate/Transition (Condition/Event-, State/Transition) Nets:
  - Places represent conditions, states, or predicates
  - Transitions represent the firing of events:
    - if a transition has one input place, the event fires immediately if a token arrives in that place
    - If a transition has several input places, the event fires when all input places have tokens
- A transition has *input* and *output* places (pre- and postconditions)
  - The presence of a token in a place is interpreted as the condition is true



• A PN is a 5-tuple,  $P = (P, T, F, W, M_0)$  with

 $\vec{P} = \{ p_1, p_2, \dots, p_m \}$   $T = \{ t_1, t_2, \dots, t_m \}$   $F \subseteq (P \times T) \cup (T \times P)$   $W : F \rightarrow \{ 1, 2, 3, \dots \}$   $M_0 : P \rightarrow \{ 0, 1, 2, 3, \dots \}$   $P \cap T = \emptyset, P \cup T \neq \emptyset$ 

is a finite set of places,

is a finite set of transitions,

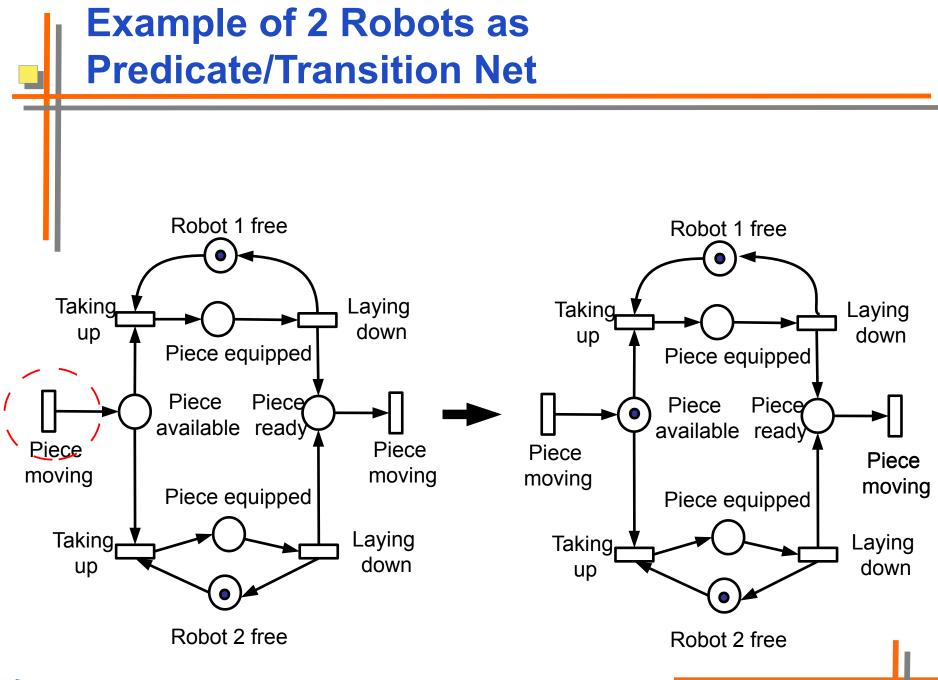
is a set of arcs (flow relation),

is a weight function,

is the initial marking, (if img(P) = {0,1}, we have a elementary net, otherwise an integer net)

A PN structure N = (P, T, W) without any specific initial marking is denoted N A PN with the given initial marking is denoted by (N,  $M_0$ )

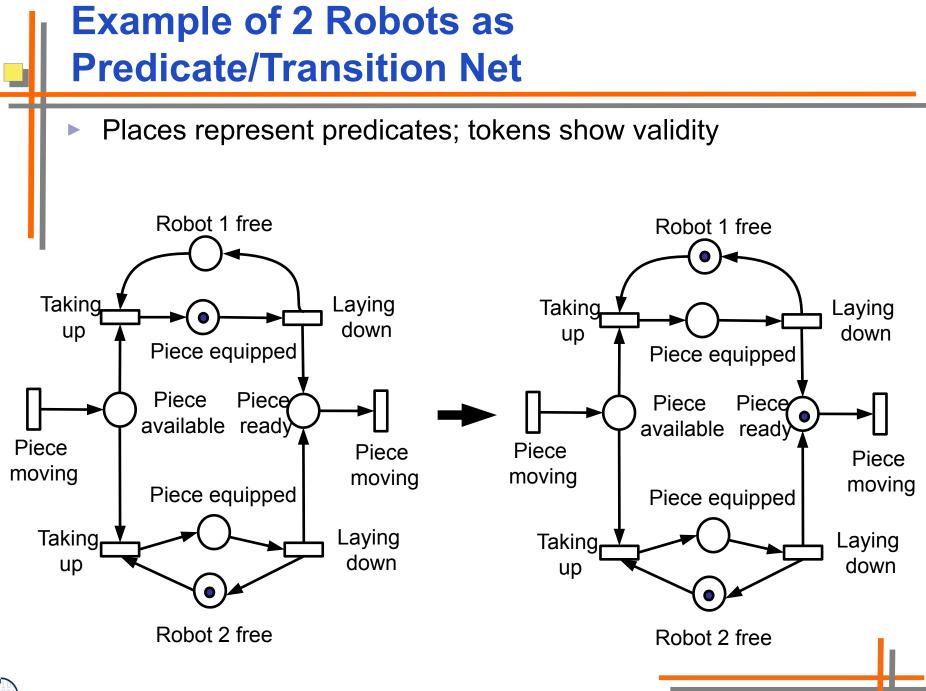






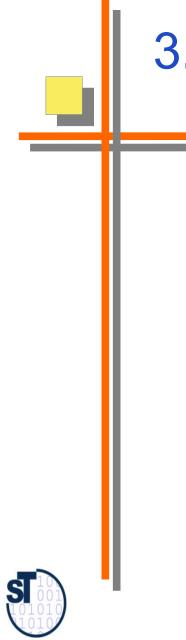
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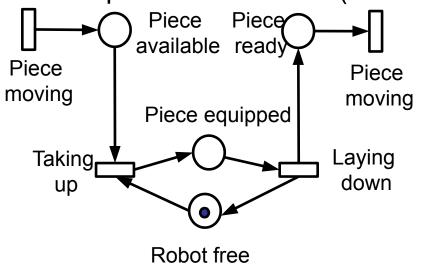
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### 3.1.2 Special Nets

### Marked Graphs (MG, Data-Flow Graphs, Data-Flow Diagrams, DFD)

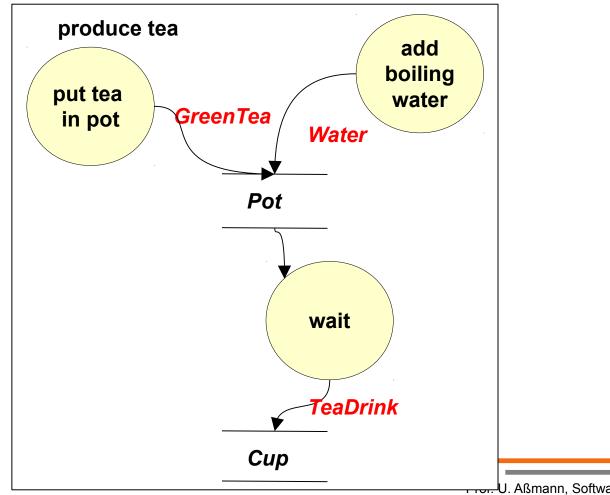
- A Marked Graph (MG) is an elementary PN such each place is the input to only one transition and the output of only one transition
  - Marked Graphs are Data-flow graphs (Data flow diagrams, DFD)
  - Transitions correspond to processes in DFD, places to stores
  - States can be *merged* with the ingoing and outcoming  $\operatorname{arcs} \rightarrow \mathsf{DFD}$
- All theory for CPN holds for DFD, too [BrozaWeide]
  - Bsp. Robot is a DFD (but not the assembly line):





### For DFD, Many Notations Exist

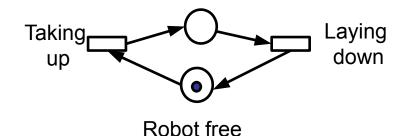
Notation from Structured Analysis [Balzert]

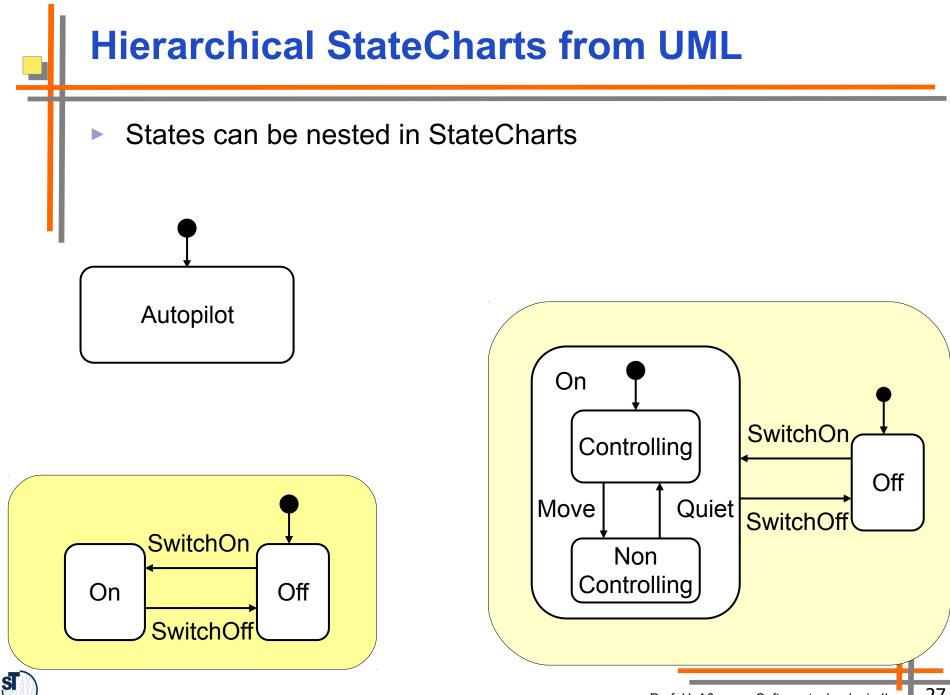




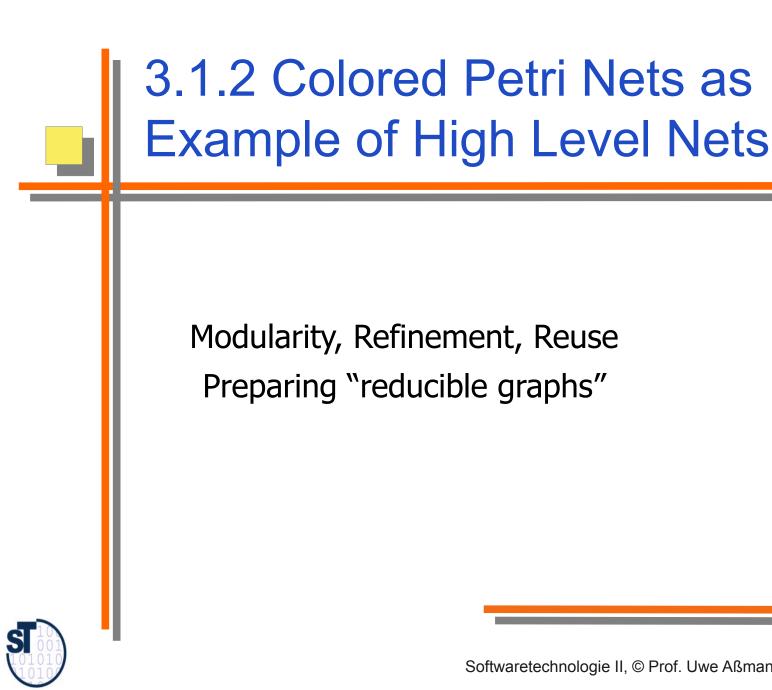
### State Machines are PN with Cardinality Restrictions

- A Finite State Machine PN is an elementary PN such that each transition has only one input and one output place
  - Then, it is equivalent to a finite automaton or a *statechart*
  - From every class-statechart that specifies the behavior of a class, a State Machine can be produced easily
    - · Flattening the nested states
  - Transitions correspond to transitions in statecharts, states to states
  - Transitions can be *merged* with the ingoing and outcoming arcs
  - In a FSM there is only one token
- All theory for CPN holds for Statecharts, too
- Ex. Robot is an FSM (but not with incoming data flow):





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### **Colored Petri Nets, CPN**

- Colored (Typed) Petri Nets (CPN) refine Petri nets:
  - Tokens are typed (colored)
  - Types are described by data structure language, such as Java, ML, UML class diagrams
  - but may also be data dictionaries, grammars
  - Concept of time can be added
- Full tool support
  - Fully automated code generation in Java and ML (in contrast to UML), e.g., DesignCPN of Aarhus University http://www.daimi.aau.dk
  - Prover proofs features about the PN
  - Net simulator allows for debugging
- Much better for safety-critical systems than UML, because proofs can be done



### **Annotations in CPN**

- Places are annotated by
  - Token types
    - . (STRING x STRING)
  - Markings of objects and the cardinality in which they occur:
    - . 2'("Uwe","Assmann")
- Edges are annotated by
  - Type variables which are unified by unification against the token objects
    - . (X,Y)
  - Guards
    - . [ X == 10]
  - if-then-else statements

. if X < 20 then Y := 4 else Y := 7

- switch statements
- boolean functions that test conditions

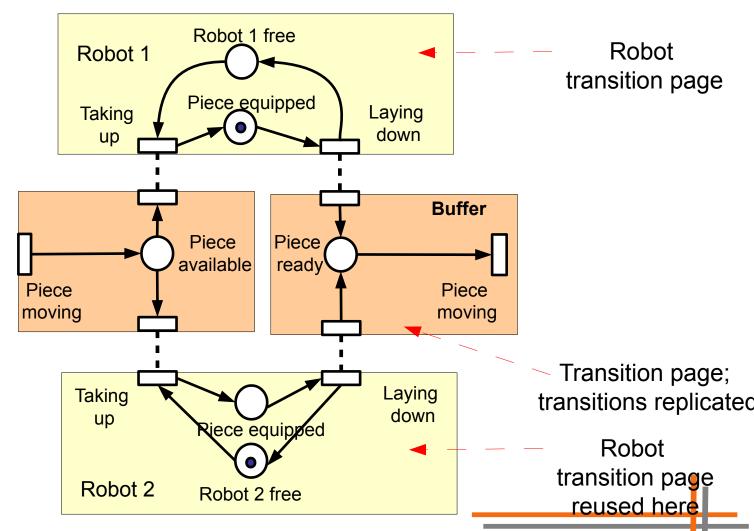


### **CPN are Modular**

- A subnet is called a *page (module)* 
  - Every page has ports which mark in- and out-going transitions (into a place) or in- and outgoing places (into a transition)
- Transition page: interface contains transitions (transition ports)
- Place page (state page): interface contains place (place ports)
- Net class: a named page that is a kind of "template" or "class"
  - It can be instantiated to a net "object"
- Reuse of pages and templates possible
  - Libraries of CPN "procedures" possible



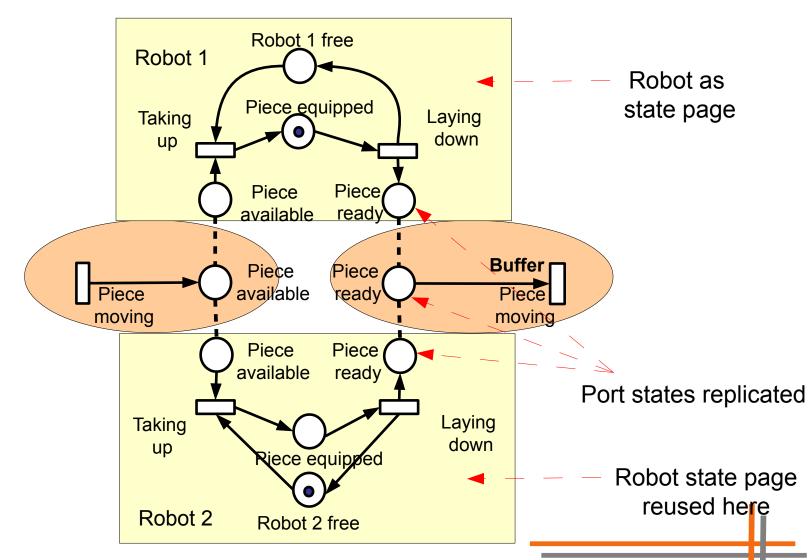
## Robots with Transition Pages, Coupled by Transition Ports





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#### Robots with Place (State) Pages, Coupled by Replicated State Ports





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### **CPN are Hierarchical**

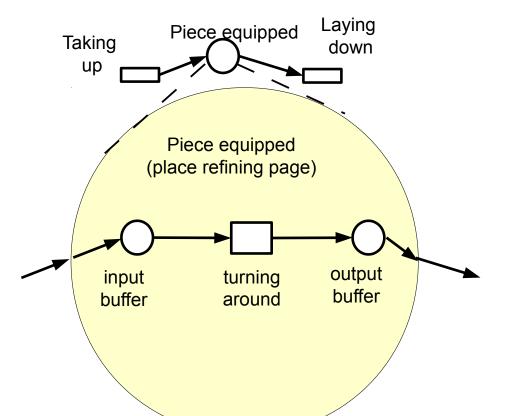
- Places and transitions may be hierarchically refined
  - Two pointwise refinement operations:
    - Replace a transition with a transition page
    - Replace a state with a state page
  - Refinment condition: Retain the embedding (embedding edges)
- CPN can be arranged as hierarchical graphs (reducible graphs, see later)
  - Large specifications possible, overview is still good
  - Subnet stemming from refinements are also place or transition pages



### **Point-wise Refinement Example**

#### Pointwise refinement:

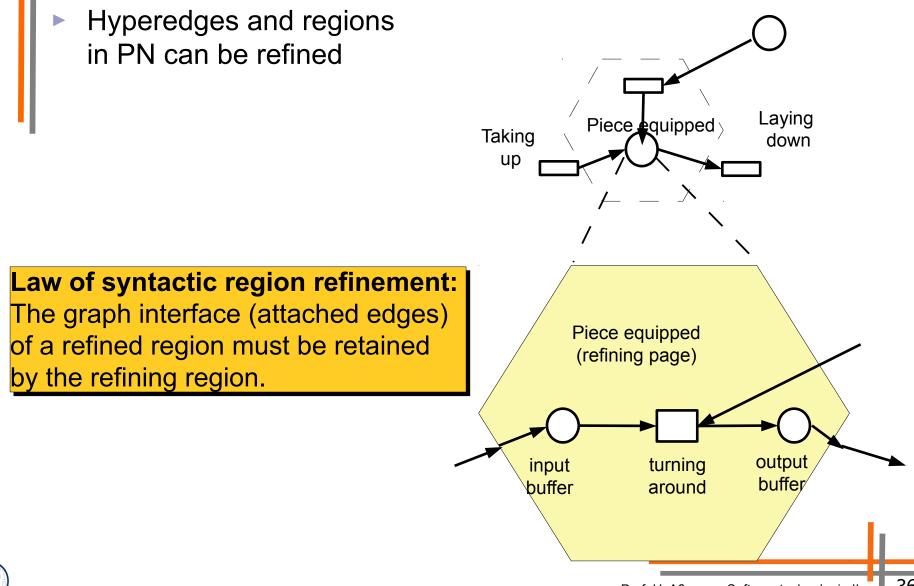
- Transition refining page: refines a transition, transition ports
- Place refining page (state refining page): refines a place, place ports



Law of syntactic refinement: The graph interface (attached edges) of a refined node must be retained by the refining page.



### **Region (Hyperedge) Refinement Example**



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### **Industrial Applications of CPN**

- Large systems are constructed as reducible specifications
- ..have 10-100 pages, up to 1000 transitions, 100 token types
- Example: ISDN Protocol specification
  - Some page templates have more than 100 uses
  - Corresponds to millions of places and transitions in the expanded, non-hierarchical net
  - Can be done in several person weeks



## 3.2 Patterns in Petri Nets

Analyzability:

# Petri Nets can be analyzed for patterns (by pattern matching)

# Modelling of Parallelism and Synchronization

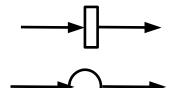
Petri Nets have a real advantage when parallel processes and synchronization must be modelled Many concepts can be expressed as *PN patterns* 



### **Simple PN Buffering Patterns**

Permanently live transition generating objects (object source) Reservoir Place (does not generate objects) Permanently live transition deleting/consuming objects (object sink)

Archive of objects

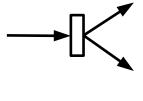


Process; sequentialization; action

Intermediate archive (buffer)

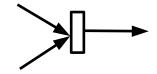


### **Parallelism Patterns**

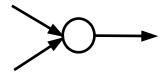


Replication and distribution of objects; forking off parallelism

Forking off parallelism Joining parallelism synchronization barrier

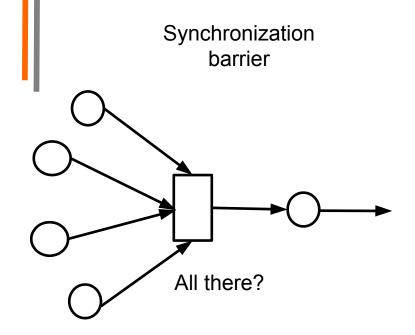


Collecting objects from parallel processes (join)

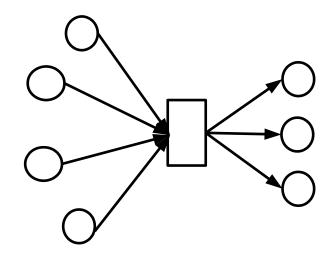




### **Examples for Building Blocks**

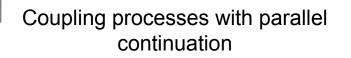


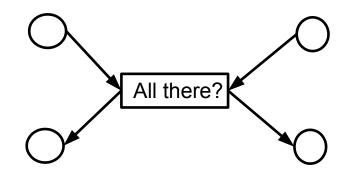
Bridges: Transitions between phases



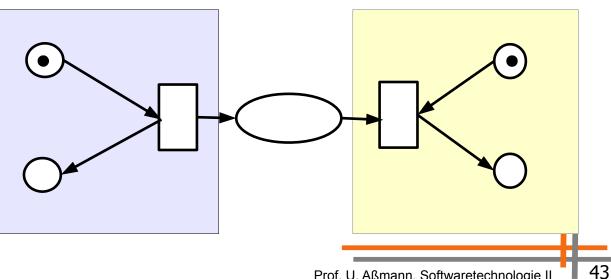


### **Patterns for Parallelism**





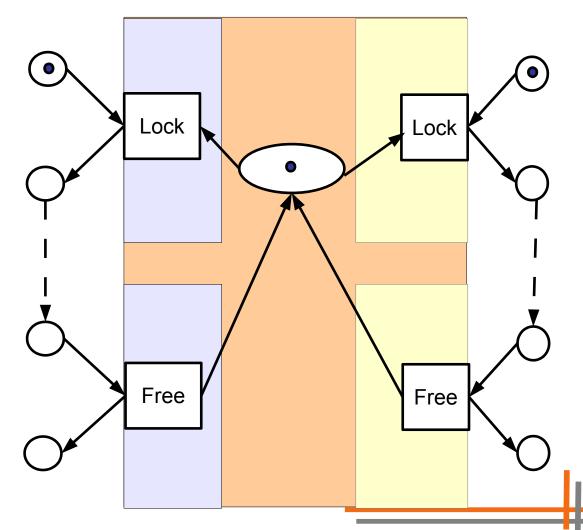
Producer/Consumer with buffer (CSP channel)





### **Semaphores For Mutual Exclusion**

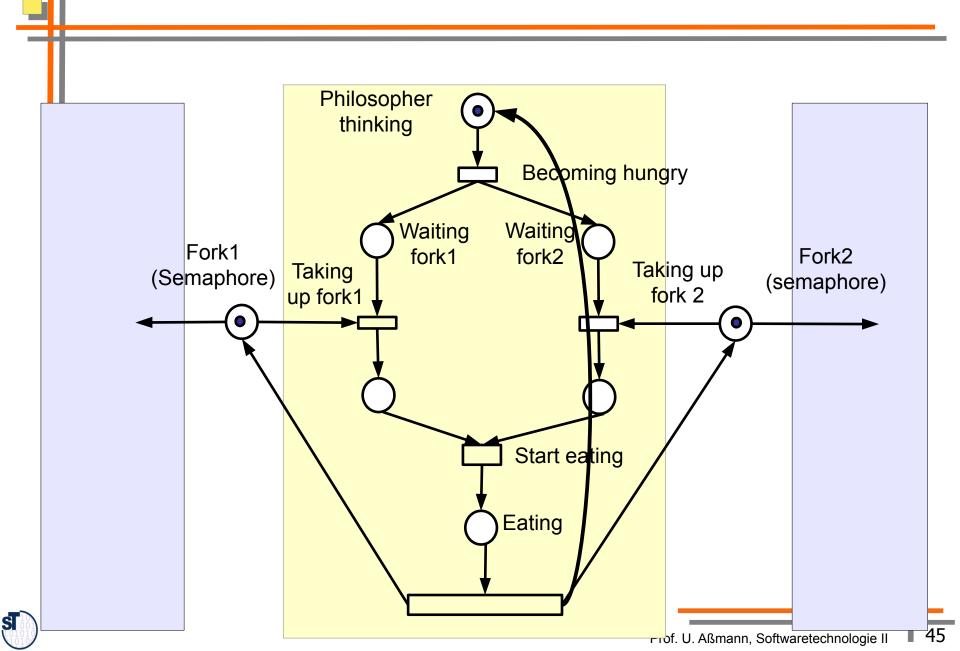
Binary or counting semaphores: depends on the capacity of the semaphore place





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### **Dining Philosophers**



### **Advantage**

- Patterns can be used to model specific requirements
- PN can be checked for patterns by Pattern Matching (Graph Rewriting)
  - Patterns can be restructured (refactorings)
  - Patterns can be composed (composition)
- Further semantic analysis of PN: Parallel, indeterministic systems can be checked for
  - Absence of deadlocks: will the parallel system run without getting stuck?
  - Liveness: will all parts of the system work forever?
  - Fairness: will all parts of the system be loaded equally?
  - Bounded resources: will the system use limited memory, and how much? (important for embedded systems)
  - Whether predicates hold in certain states (model checking)

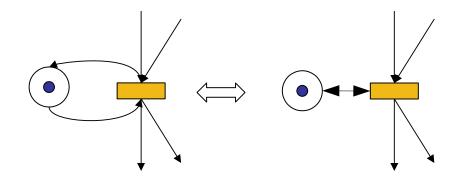


# 3.3 Refactorings (Reduction Rules) for Petri Nets ... in the form of graph rewrite rules Softwaretechnologie II, © Prof. Uwe Aßmann

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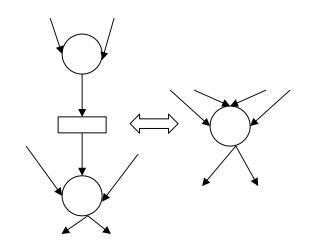
### Special Restructuring Patterns (Refactorings)

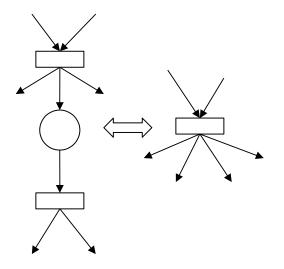
- Source transitions are always enabled, i.e., generate tokens (token generator)
- Sink transitions are always enabled and swallow tokens (token sink)
- A self-loop is a pair of a place p and a transition t if p is both output and input place of t
  - A PN without any self-loops is *pure*. Its arc relation is irreflexive





### **Simple Reduction Rules**



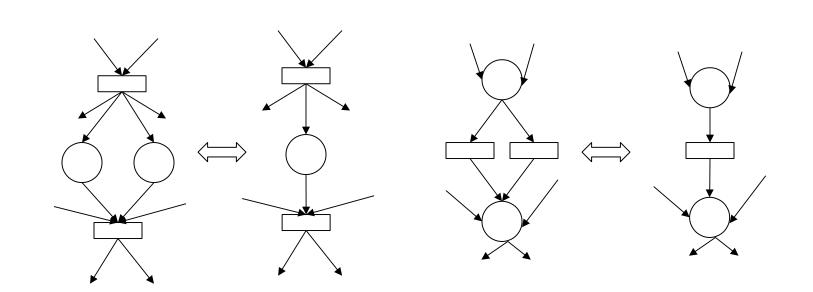


1) Fusion of Series Places (FSP) (Bridge elimination)

2) Fusion of Series Transitions (FST) (Intermediate buffer elimination)



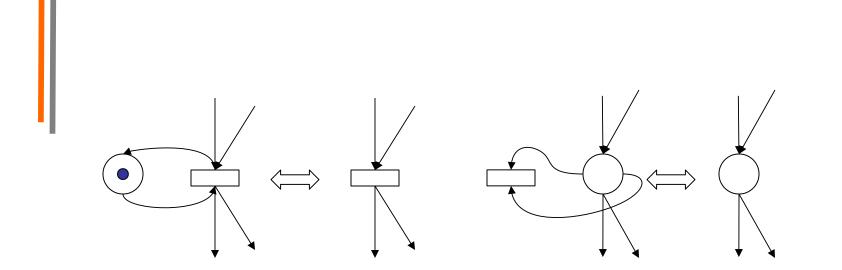
### **Simple Reduction Rules**



3) Fusion of Parallel Places (FPP) 4) Fusion of Parallel Transitions (FPT)



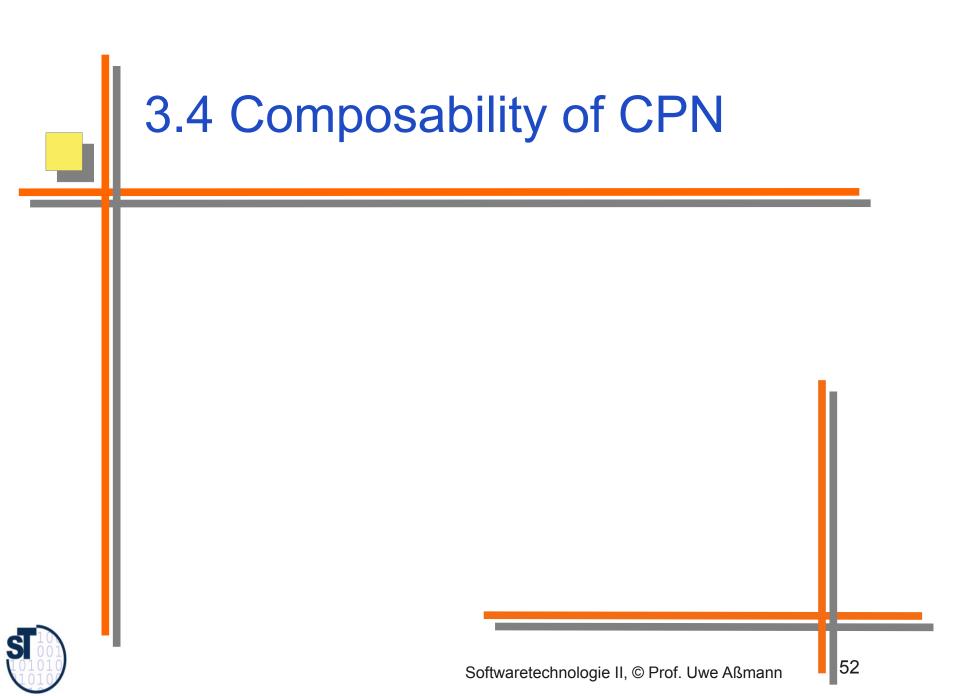
### **Simple Reduction Rules**



5) Elimination of Self-loop Places (ESP) 6) Elimination of Self-loop Transitions (EST)

All transformations preserve liveness, safeness and boundedness.





### Case Study for Composition: Pervasive Healthcare Middleware (PHM)

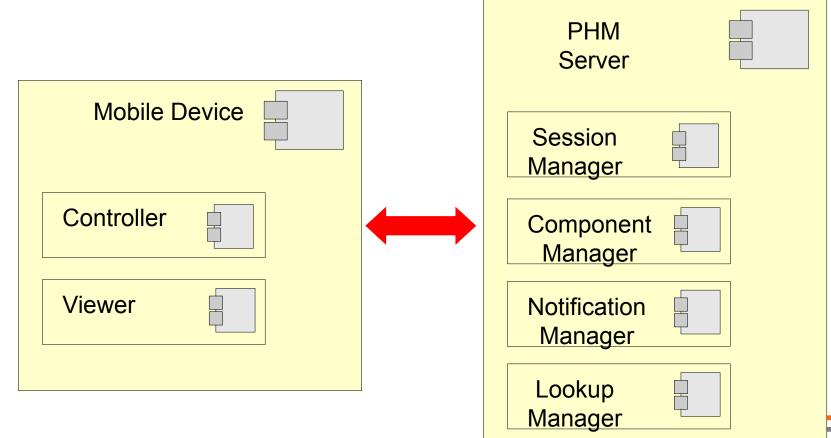
- in development at the Pervasive Computing Center, University of Aarhus
- Basic idea:
  - Specify the structure of an application with UML
  - and the behavior with CPN, describing the behavior of the classes/objects (object lifecycle)
  - Glue behavior together with page glueing mechanism
- Electronic patient records (EPR) replace the papers
  - First version in 2004, on stationary PC
  - Next versions for pervasive computing (PDA, wireless):
    - Hospital employees will have access to the patient's data whereever they go, from Xray to station to laboratories
  - For instance, medication plans are available immediately



### **The PHM Architecture**

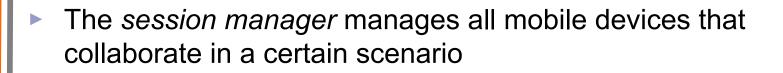
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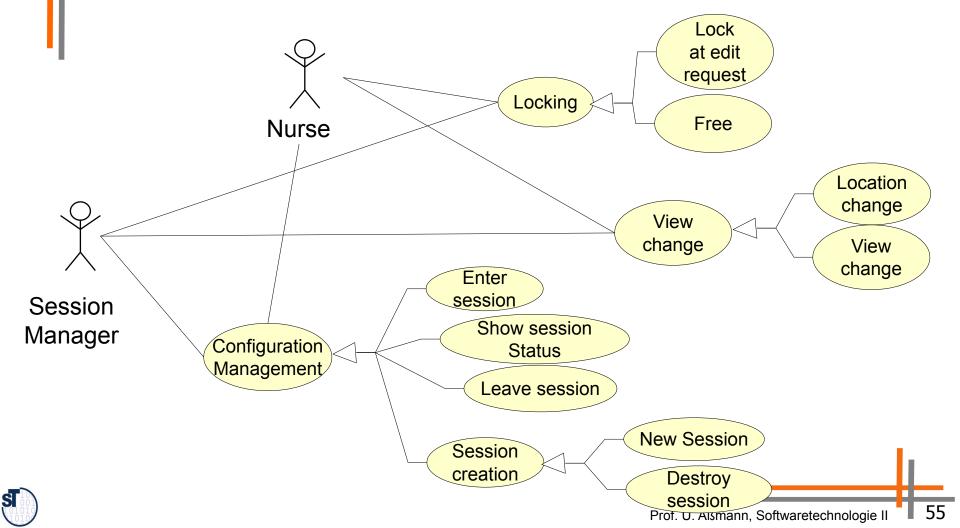
A session is entered by several mobile devices that collaborate



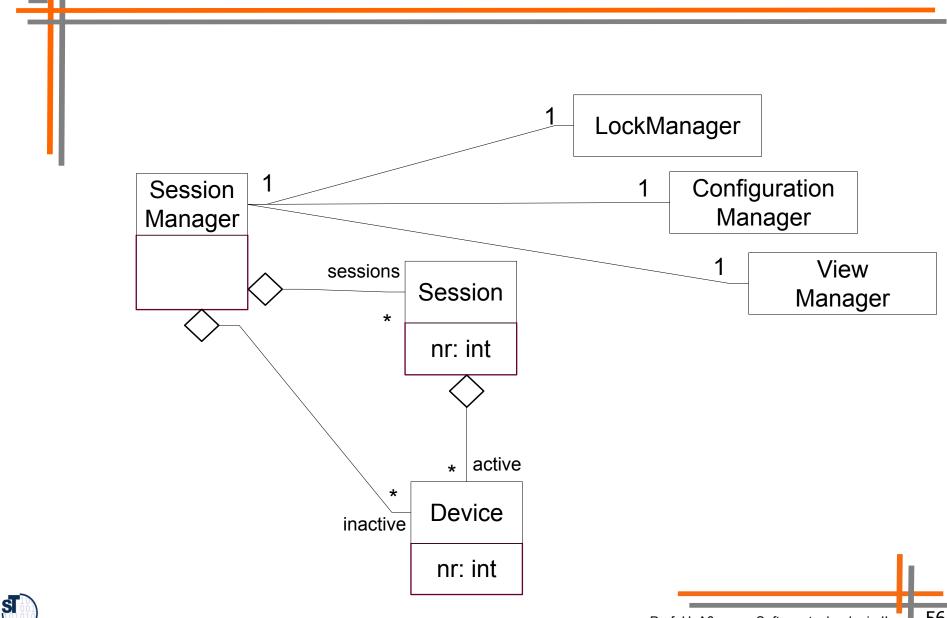
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### **Session Manager Use Cases**

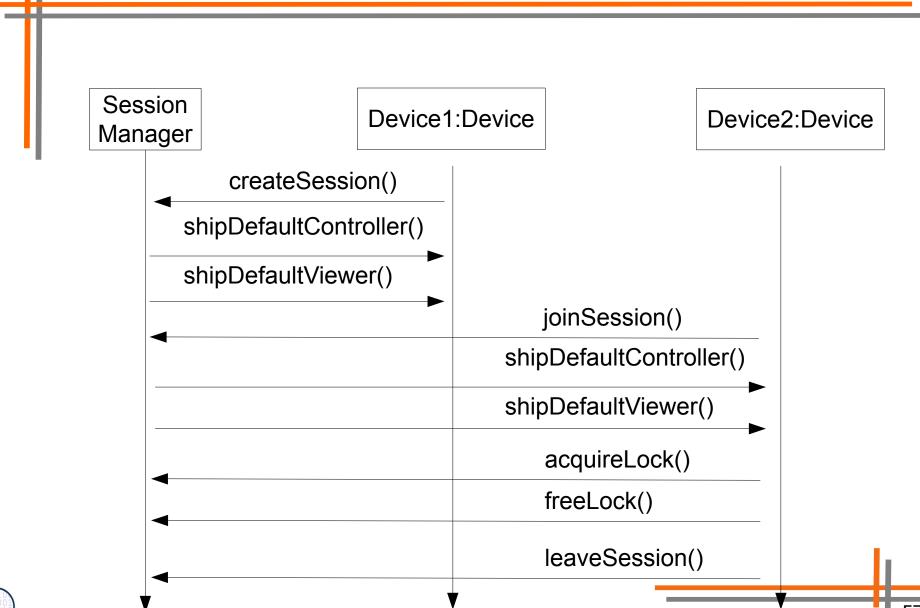




### **Class Diagram Session Manager**



### **Sequence Diagram Session Manager**



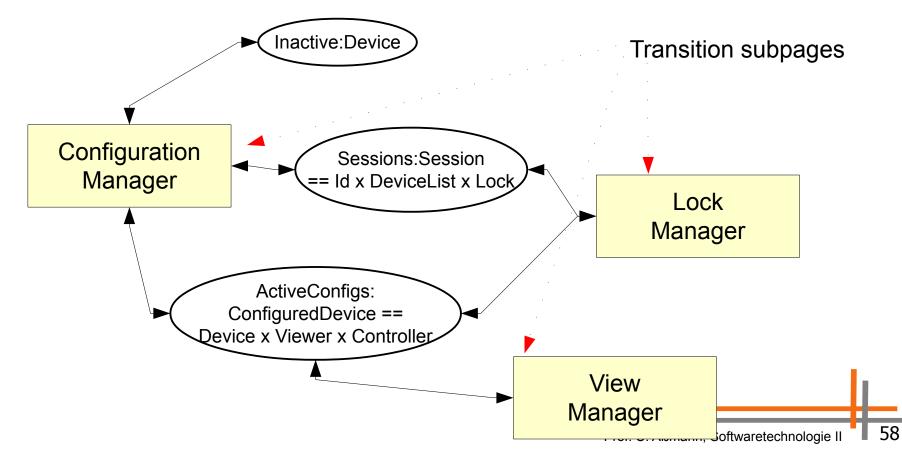
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### **Session Manager Top-Level CPN**

- Double arrows indicate that arrows run in both directions
- Basic Types

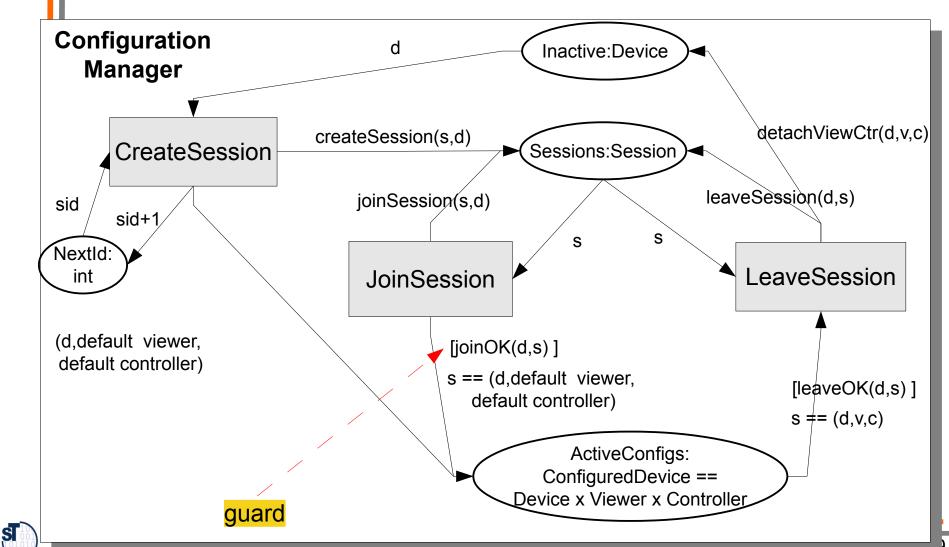
S

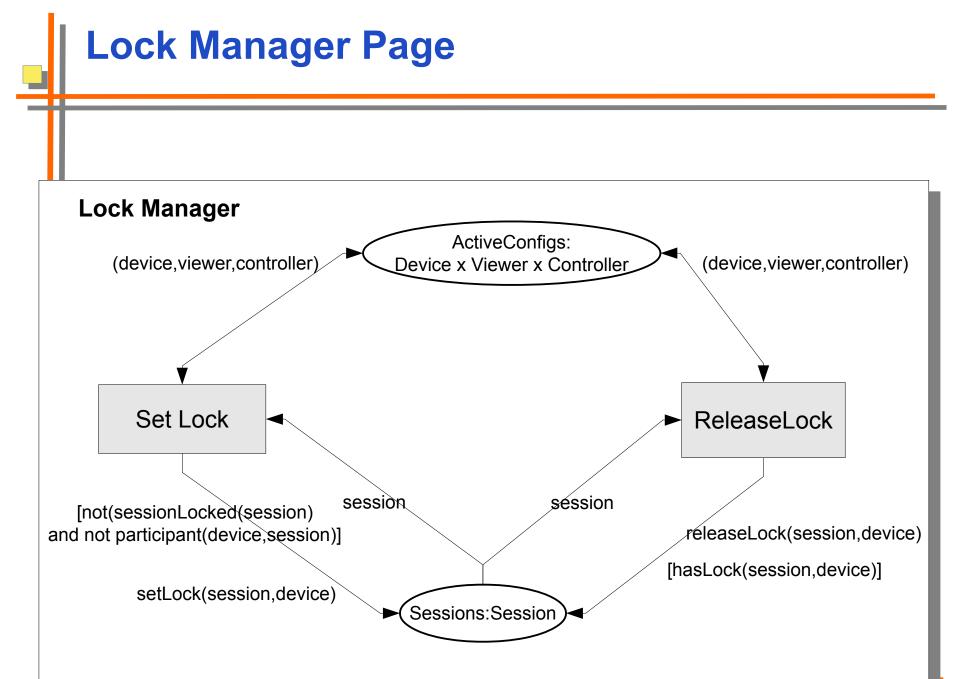
- Session ::= SessionId DeviceList LockType
- ConfiguredDevice ::= Device Viewer Controller



### **Configuration Manager Page**

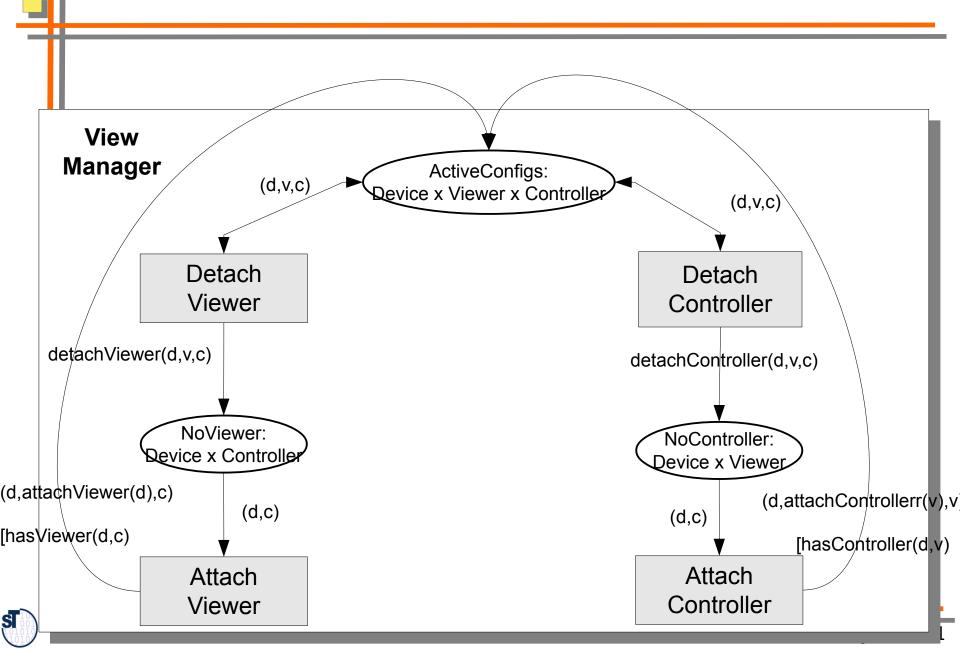






### ST.

### **View Manager Page**





- The CPN pages are attached to UML classes, i.e., describe their behavior
  - States and transitions are marked by UML types
- Every subpage is coupled to others
  - via common states (port or join states)
    - The union of the pages via join states is steered by OR, i.e., the pages add behavior, but do not destroy behavior of other pages
  - Via common transitions (*port* or *join transitions*)
    - The union of the pages via join transitions is steered by AND, i.e., the pages add behavior and synchronize with transitions of other pages
- Transitions are interpreted as coarse-grain events
  - On the edges, other functions (actions) are called
  - Hence, CPN are open: if something is too complicated to model as a PN, put it into functions

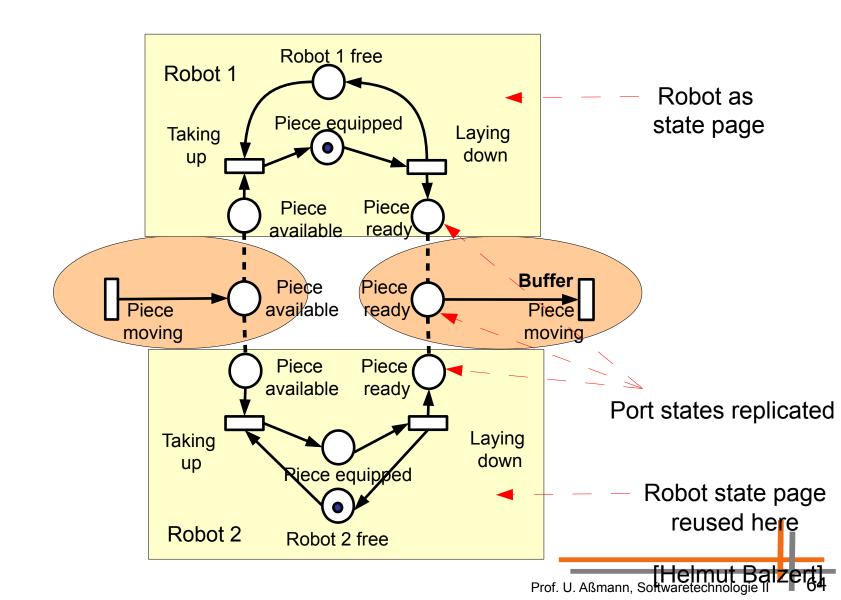


### **Coupling of Place and Transition Pages**

- Port state coupling (or fuse, merge, composition): Place pages are coupled to other place pages via common states (port states)
  - The union of the pages is steered by OR, i.e., the pages add behavior, but do not destroy behavior of other pages
- Port transition coupling: Transition pages are coupled to other transition pages via common transitions (port transitions)
  - The union of the pages is steered by AND, and every page changes the behavior of other page
  - Events must be available on every incoming edge of a transition
  - The transitions of the combined net only fire if the transitions of the page components fire

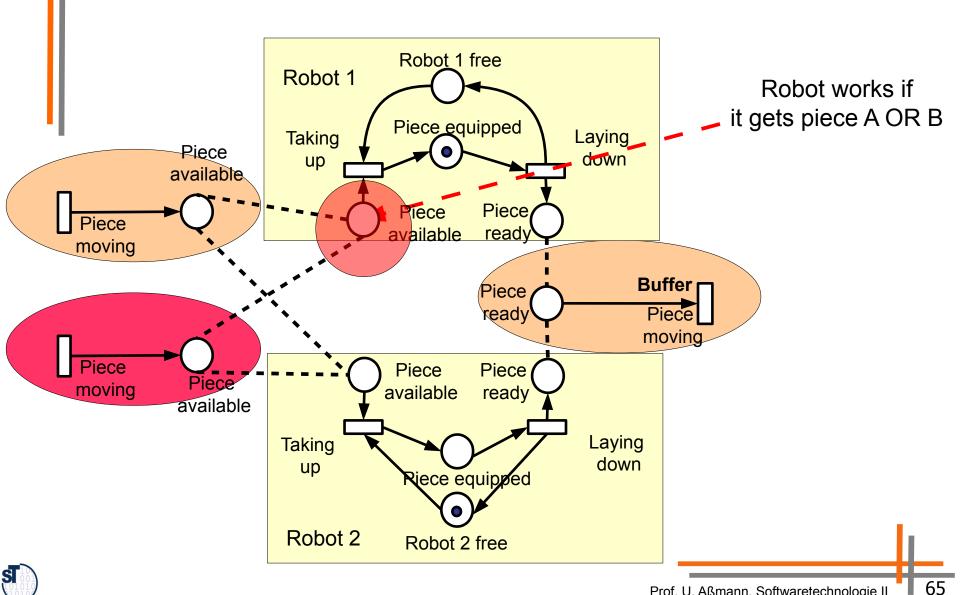


# Robots with State Pages, Coupled by Replicated State Ports

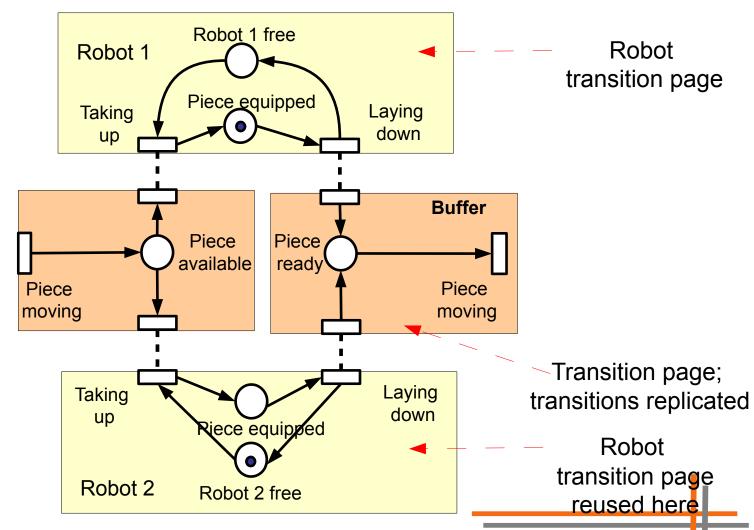




### **A Robot OR-composed View**



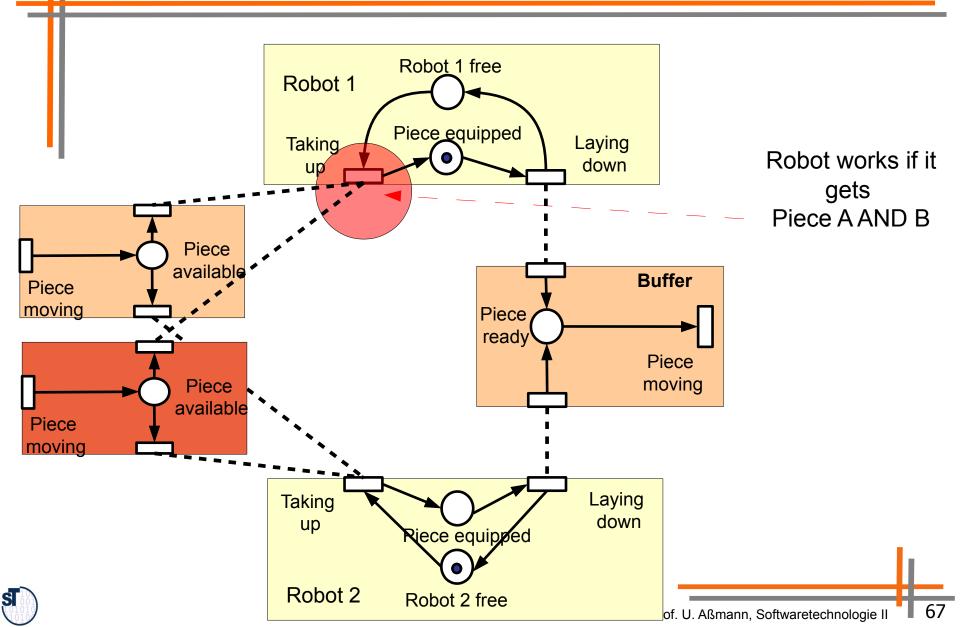
# Robots with Transition Pages, Coupled by Transition Ports





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### **A Robot AND-composed view**



### **Advantages of CPN for the PHM**

- The PHM is a distributed and mobile scenario
  - Devices can fail (battery empty, wireless broken, etc)
  - The resulting CPN can be checked on deadlock, i.e., will the PHM session manager get stuck?
- Compact specification
  - Usually, CPN are much more compact than statecharts
- Variability
  - The pages are modular, i.e., can be exchanged for variants easily (e.g., other locking scheme)



# 3.4 Parallel Composition of Colored Petri Nets



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### **Parallel composition of PN**

- Complex synchronization protocols can be abstracted to a pattern (als called transition page or a place page)
- When joining PN with AND (i.e., joining transition pages), synchronization protocols can be overlayed to existing sequential specifications

### **Unforeseeable Extensible Workflows**

- Workflows are described by Colored Petri Nets (CPN) or languages built on top of CPN:
  - YAWL language [van der Aalst]
  - Workflow nets
- We can use the extension of CPN for workflow composition, enriching a workflow core with a workflow aspect:
  - Place extension (State extension): adding more edges in and out of a place (state):
    - OR-based composition: Core OR view: Core-place is ORed with Aspect-Place
  - Transition extension (Activity extension): adding more edges in and out of a transition (activity)
    - AND-based composition: Core-transition is ANDed with Aspect-transition

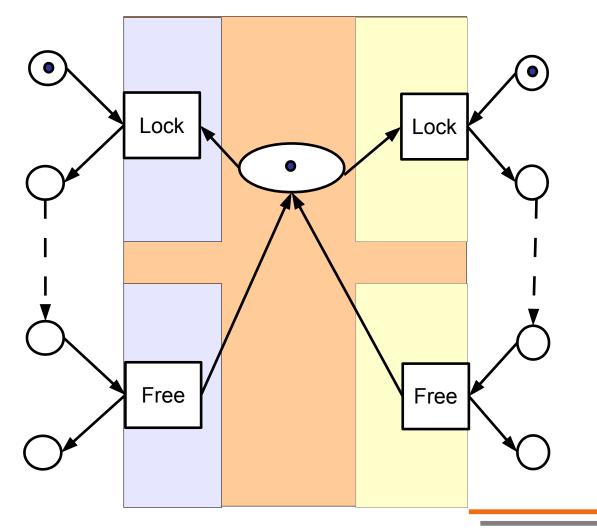


### Weaving Patterns for Synchronization Protocols with AND Composition

Complex synchronization protocols can be abstracted to a transition page
Weaving them with AND, they can be overlayed to existing sequential specifications

### **Semaphores For Mutual Exclusion Revisited**

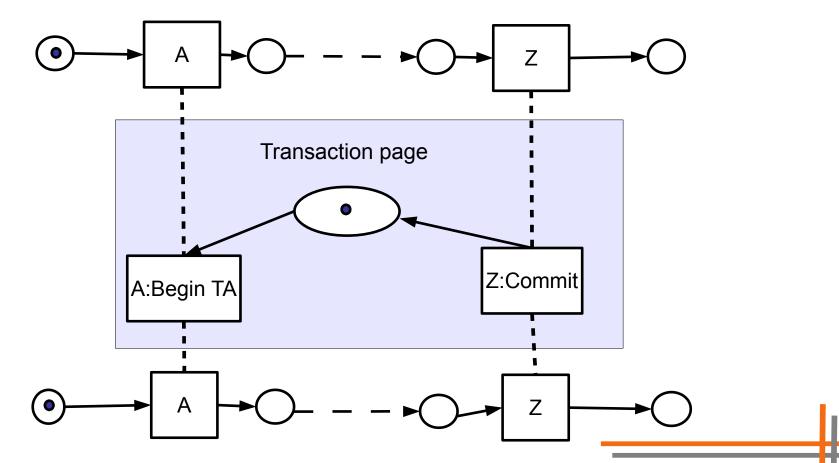
Forms a synchronisation aspect via ANDed Lock transitions





### **Transaction Protocols as AND-Aspects**

 Crosscut between processes (cores) and transaction protocol (aspect)





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### Insight

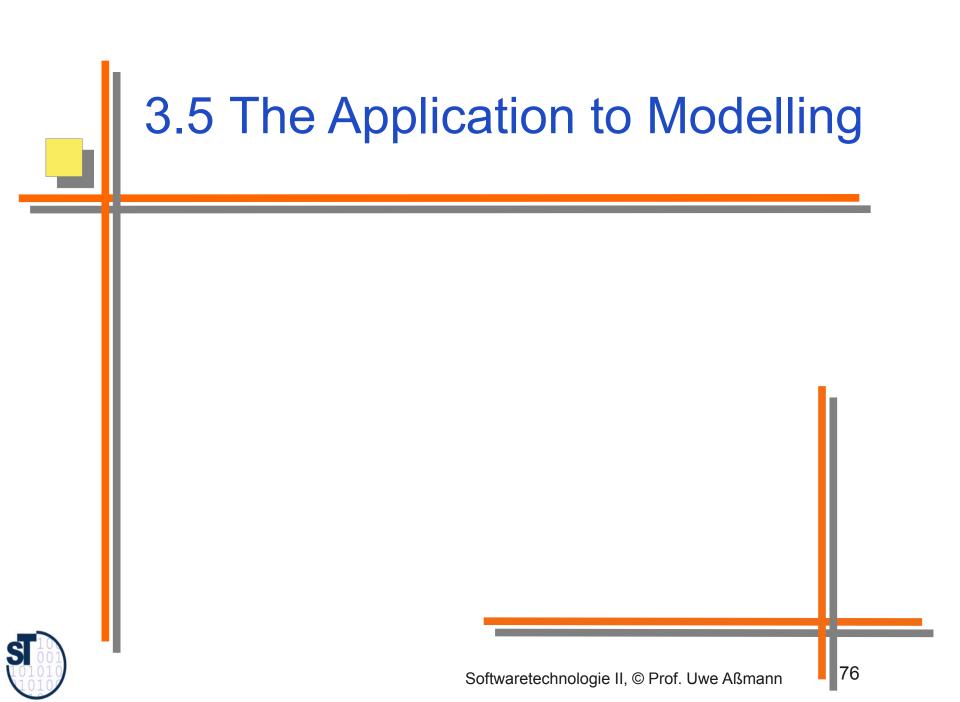
AND-Merge and OR-Merge of CPN are sufficient basic composition operators for building complex aspect weavers for workflow languages built on CPN

#### AND-weaving for synchronization

#### OR-weaving for functional extension



Prof. U. Aßmann, Softwaretechnologie II



### Petri Nets Generalize UML Behavioral Diagrams

#### **Activity Diagrams**

- Activity Diagrams are similar to PN, but not formally grounded
  - Without markings
  - No liveness analysis
  - No resource consumption analysis with boundness
  - No correspondence to UML statechart, although for PN holds that PN with finite reachability graphs correspond to finite automata
- I.e., it is difficult to prove something about activity diagrams, and difficult to generate (parallel) code from them.

### **Data-flow diagrams**

 DFD are special form of activity diagrams, and correspond to Marked Graphs

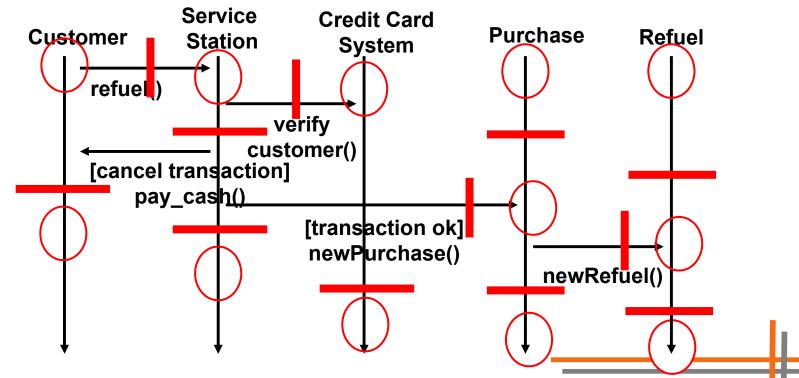
#### **Statecharts**

- Finite automata are restricted form of Petri nets
- The hierarchical structuring in Statecharts is available in High-Level Petri Nets (e.g., CPN)



### Petri Nets Generalize UML Sequence Diagrams

- The object life lines of a sequence diagram can be grouped into state such that a PN results
- All of a sudden, liveness conditions can be studied
  - Is there a deadlock in the sequence diagram?
  - Are objects treated fair?





### A Simple Modelling Process for Safety-Critical Software with CPN

- Elaboration: Identify active and passive parts of the system
  - Active become transitions, passive to places
- Elaboration: Find the relations between places and transitions
- Elaboration: How should the tokens look like: boolean? Integers? Structured data?
  - Use UML class diagrams as token type model
- Restructure: Group out subnets to separate "pages"
- Refactor: Simplify by reduction rules
- Elaboration: Analyse the specification on liveness, boundedness, reachability graphs, fairness. Use a model checker to verify the CPN
- TransformRepresentation: Produce views as statecharts, sequence, collaboration, and activity diagrams..



### How to Solve the Reactor Software Problem?

- Specify with UML and CPN
  - Verify it with a model checker
  - Let a prototype be generated
  - Test it
  - Freeze the assembler
- Then, verify the assembler, because you should not trust the CPN tool nor the compiler
  - Any certification agency in the world will require a proof of the assembler!
- However, this is much simpler than programming reactors by hand...



### **The Gloomy Future of PN**

- PN will become the major tool in a future CASE tool or integrated development environment
  - Different views on the PN: state chart view, sequence view, activity view, collaboration view!
- Many isolated tools for PN exist, and the world waits for a full integration into UML
- CPN will be applied in scenarios where parallelism is required
  - Architectural languages
  - Web service langauges (BPEL, BPMN, ...)
  - Workflow languages
  - Coordination languages





 Thanks to Björn Svensson for help in making slides, summarizing [Murata]