

2. Modelling Dynamic Behavior with Petri Nets

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- 1. Basics
 - 1. Elementary Nets
 - 2. Special Nets
 - 3. Colored Petri Nets
- 2. Patterns in Petri Nets
- 3. Application to modelling

Obligatory Readings

- Balzert et al. (german)
 - Chapter 10.4 (p. 303ff)
- Ghezzi et al. (english)
 - Chapter 5.5.4 (p. 185ff)
- <u>http://www.scholarpedia.org/article/Petri_net</u>



Secondary Literature

- W.M.P. van der Aalst and A.H.M. ter Hofstede. <u>Verification of workflow</u> <u>task structures: A petri-net-based approach</u>. Information Systems, 25(1): 43-69, 2000.
- Kurt Jensen, Lars Michael Kristensen and Lisa Wells. <u>Coloured Petri Nets</u> <u>and CPN Tools for Modelling and Validation of Concurrent Systems</u>. Software Tools for Technology Transfer (STTT). Vol. 9, Number 3-4, pp. 213-254, 2007.
- J. B. Jörgensen. <u>Colored Petri Nets in UML-based Software</u> <u>Development – Designing Middleware for Pervasive Healthcare</u>. www.pervasive.dk/publications/files/CPN02.pdf
- Web portal "Petri Net World" <u>http://www.informatik.uni-hamburg.de/TGI/PetriNets</u>



Further Literature

- K. Jensen and L. M. Kristensen. <u>Colored Petri Nets</u>. Springer, 2009. (<u>http://cs.au.dk/~cpnbook/</u>)
- T. Murata. Petri Nets: properties, analysis, applications. IEEE volume 77, No 4, 1989.
- W. Reisig. <u>Elements of Distributed Algorithms Modelling and</u> <u>Analysis with Petri Nets.</u> Springer. 1998.
- W. Reisig, G. Rozenberg. <u>Lectures on Petri Nets I+II</u>, Lecture Notes in Computer Science, 1491+1492, Springer.
- > J. Peterson. **Petri Nets**. ACM Computing Surveys, Vol 9, No 3, Sept 1977



Goals

- Understand <u>Untyped</u> (Page/Transition nets) and <u>Colored Petri nets</u> (CPN)
- Understand that PN/CPN are a verifiable and automated technology for safety-critical systems
- Understand why PN are a good modeling language for parallel systems simulating the real world
- > PN have subclasses corresponding to finite automata and data-flow graphs
- > PN can be refined, then reducible graphs result



The Initial Problem

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You work for PowerPlant Inc. Your boss comes in and says: "Our government wants a new EPR reactor, similarly, in the way Finland has it."How can we produce a verified control software? We need a good modelling language!



How do we produce software for safety-critical systems?



Projects with Safety-Critical, Parallel Embedded Software

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Aerospace

 The WITAS UAV unmanned autonomously flying helicopter from Linköping http://www.ida.liu.se/~marwz/papers/ICAPS06_System_Demo.pdf

Automotive

 Prometheus: driving in car queues on the motorway http://www.springerlink.com/content/j06n312r36805683/

Trains

- <u>www.railcab.de</u> Autonomous rail cabs
- The Copenhagen metro (fully autonomous)
 - Inauguration seminar <u>http://www.cowi.com.pl/SiteCollectionDocuments/cowi/en/menu/02.%20Serv</u> <u>ices/03.%20Transport/5.%20Tunnels/Other%20file%20types/Copenhagen%2</u> <u>0Metro%20Inauguration%20Seminar.pdf</u>





3.1 Basics of PN

Petri Net Classes

- Predicate/Transition Nets: simple tokens, no hierarchy.
- Place-Transition Nets: multiple tokens
- High Level Nets: structured tokens, hierarchy
- There are many other variants, e.g., with timing constraints

Petri Nets

Model introduced by Carl Adam Petri in 1962,

C.A. Petri. Ph.D. Thesis: "Communication with Automata".

- Over many years developed within GMD (now Fraunhofer, FhG)
- PNs specify diagrammatically:
 - Infinite state systems, regular and non-decidable
 - Concurrency (parallelism) with conflict/non-deterministic choice
 - Distributed memory ("places" can be distributed)
- Modeling of parallelism and synchronization
- Behavioral modeling, state modeling etc.



- Tupel (P,T,F,W,m₀)
 - **P** = Places $P \cap T = \emptyset$
 - T = Transistions
 - **F** = Flow Relations $F \subseteq (P \times T) \cup (T \times P)$
 - **W** = (Relation) Weight $W: F \to \mathbb{N}_0$ wobei
- $$\begin{split} W \colon F \to \mathbb{N}_0 \text{ wobei} \\ W(\mathbf{p}, \mathbf{t}) &= 0 \ \equiv (p, t) \notin F, p \ \in P \text{ und } t \ \in T \text{ und} \\ W(\mathbf{t}, \mathbf{p}) &= 0 \ \equiv (t, p) \notin F, p \ \in P \text{ und } t \ \in T \end{split}$$

•
$$\mathbf{m}_0$$
 = Start Marking

$$m_0: P \rightarrow \mathbb{N}_0$$





- ▶ A marking $m(p) \rightarrow \mathbb{N}_0$, $p \in P$ assigns a non-negative Integer to places
 - Number of tokens in a place
- ▶ A weight $W(f) \rightarrow \mathbb{N}_0$, $f \in F$ assigns a non-negative Integer to arcs
 - How many tokens can they carry





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• Transition $t \in T$ is **enabled** when

 $m(p) - W(p,t) > 0, \forall p \in P$

For all incoming arcs, the places must contain at least n tokens
→ n = the weight of the incoming arc





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- When a transition is Enabled, it may or may not fire
- ▶ When a transition $t \in T$ fires $m(p) = m(p) - W(p,t), \forall p \in P$
 - N Tokens are removed from all incoming places

 $m(p) = m(p) + W(t, p), \forall p \in P$

- M Tokens are added to all outgoing places
- ► The state (marking) of the Petri Net is changed





Ex.: Department of a Train





Elementary Nets: Predicate/Transition Nets

- > A **Petri Net (PN)** is a <u>directed</u>, <u>bipartite graph</u> over two kinds of *nodes*
 - 1. Places (circles)
 - 2. Transitions (bars or boxes)
- > A **Integer PN** is a <u>directed</u>, <u>weighted</u>, <u>bipartite graph</u> with integer tokens
 - Places may contain several tokens
 - Places may contain a capacity (bound=k)
 - k tokens in a place indicate that k items are available



- An *Elementary PN* (boolean net, predicate/transition or condition/event nets)
 - Boolean tokens One token per place (bound of place = 1)
 - Arcs have no weights
 - Presence of a token = condition or predicate is true
 - *Firing* of a transition = from the input the output predicates are concluded
 - Thus elementary PN can represent simple forms of logic



High-Level Nets

- > A **High-Level PN** (Colored PN, CPN) allows for <u>typed places</u> and <u>typed arcs</u>
 - For types, any DDL can be used (e.g., UML-CD)
- High-level nets are modular
 - Places and transitions can be refined
 - A Colored Petri Net is a reducible graph
- > The upper layers of a reducible CPN are called *channel agency nets*
 - Places are interpreted as channels between components





Application Areas of Petri Nets

- Reliable software (quality-aware software)
 - PetriNets can be checked on deadlocks, liveness, fairness, bounded resources
- Safety-critical software that require proofs
 - Control software in embedded systems or power plants
- Hardware synthesis
 - Software/Hardware co-design
- User interface software
 - Users and system can be modeled as parallel components



Application Area I: Behavior Specifications in UML

- Instead of describing the behavior of a class with a statechart, a CPN can be used
 - Statecharts, data flow diagrams, activity diagrams are subsets of CPNs
- CPN have several advantages:
 - They model **parallel** systems (with a fixed net) naturally
 - They are compact and **modular**, they can be reducible
 - They are suitable for **aspect-oriented** composition, in particular of parallel protocols
 - They can be used to **generate code**, also for complete applications
- > Informal: for CPN, the following features can be proven
 - Liveness: The net can fire at least n times
 - Fairness: All parts of the net are equally "loaded" with activity
 - **K-boundedness**: The number of tokens, a place can contain, are bound by k
 - Deadlock: The net cannot proceed but did not terminate correctly
 - **Deadlock-freeness**: The net contains no deadlocks



Application Area II: Contract checking (Protocol Checking) for Components

- Petri Nets describe behavior of components (dynamic semantics)
 - They can be used to check whether components fit to each other
- > Problem: General fit of components is undecidable
 - The protocol of a component must be described with a decidable language
 - Due to complexity, context-free or -sensitive protocol languages are required
- > Algorithm:
 - Describe the behavior of two components with two CPN
 - Link their ports
 - Check on *liveness* of the unified CPN
 - If the unified net is not live, components will not fit to each other...
- Liveness and fairness are very important criteria in safety-critical systems





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3.1.1 Elementary Nets (Predicate/Transition Nets)

Meaning of Places and Transitions in Elementary Nets

- Predicate/Transition (Condition/Event-, State/Transition) Nets:
 - Places represent conditions, states, or predicates
 - Transitions represent the firing of events:
 - if a transition has one input place, the event fires immediately if a token arrives in that place
 - If a transition has several input places, the event fires when all input places have tokens
- A transition has input and output places (pre- and postconditions)
 - The presence of a token in a place is interpreted as the condition is true



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Robot 2 free



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Robot 2 free







Comparing PN to Automata

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Petri Nets

- Tokens encode parallel "distributed" global state
- Can be switched "distributedly"

Automata

- Sequential
- One global state (one token)
- Can only be switched "centrally"





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3.1.2 Special Nets (Special Syntactic forms of PN)

3.1.2.a Marked Graphs (MG) are DFD with Distributed Memory

- > A **Marked Graph** (MG) is a PN such that:
 - 1. Each place has only 1 incoming arc
 - 2. Each place has only 1 outgoing arc
 - Then the places can be abstracted (identified with one flow edge)
 - Transitions may split and join, however
 - No shared memories between transitions (distributed memory)
- Marked Graphs correspond to a special class of data-flow graphs (Data flow diagrams with non-shared, distributed memory, dm-DFD)
 - MG provide deterministic parallelism without confusion
 - Transitions correspond to processes in DFD, places to stores
 - States can be *merged* with the ingoing and outcoming $\operatorname{arcs} \rightarrow \mathsf{DFD}$ without stores
 - Restriction: Stores have only one producer and consumer
 - But activities can join and split
- All theory for CPN holds for marked graph DFD, too [BrozaWeide]







3.1.2.a Marked Graphs (MG)

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➢ Is the production PN a MG ?





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3.1.2.a Marked Graphs (MG)

- The production PN is no MG
 - → Some places have more than 1 incoming/outgoing arc





3.1.2.a Marked Graphs (MG)

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However, the production robot PN is a MG





More General Data-Flow Diagrams

- General DFD without restriction can be modeled by PN, too.
 - However, places cannot be abstracted
 - They correspond to stores with 2 feeding or consuming processes
- > Example: the full robot has places with 2 ingoing or outgoing edges,
 - They cannot be abstracted

For DFD, Many Notations Exist

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Notation from Structured Analysis [Balzert]





3.1.2.b State Machines are PN with Cardinality Restrictions

- > A **Finite State Machine PN** is an elementary PN such that:
 - 1. Each transition has only 1 incoming arc
 - 2. Each transition has only 1 outgoing arc
 - Then, it is equivalent to a finite automaton or a *statechart*
 - From every class-statechart that specifies the behavior of a class, a State Machine can be produced easily
 - Flattening the nested states
 - Transitions correspond to transitions in statecharts, states to states
 - Transitions can be *merged* with the ingoing and outcoming arcs
 - In a FSM there is only one token
- > All theory for CPN holds for Statecharts, too


3.1.2.b State Machines

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➢ Is the production PN a FSM ?





Robot 2 free

3.1.2.b State Machines

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- The production PN is no FSM
 - → Some transitions have more than 1 incoming/outgoing arc



3.1.2.b State Machines

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One Robot is a FSM but not with incoming/outgoing arc





Hierarchical StateCharts from UML

- States can be nested in StateCharts
- This corresponds to hierarchical StateMachine-PN, in which states can be refined and nested





3.1.2.c Free-Choice Nets

- Two transitions are in conflict if the firing of one transition deactivates another
 - R1: no conflicts (t1 and t3 activated) \rightarrow in this example t1 fires
 - R2: t2 and t3 are in conflict \rightarrow in this example t2 fires
 - R3: t3 is deactivated because of t2





3.1.2.c Free-Choice Nets

- > **Free-Choice Petri Net** provides deterministic parallelism
 - Choice between transitions never influence the rest of the system ("free choice")
 - Rule conflicts out
 - AND-splits and AND-joins
- Keep <u>places with more than one output transitions</u> away from <u>transitions</u> with more than one input places (forbidden are "side actions")
 - outdegree(place) \rightarrow in(out(place)) = {place}







3.1.3 Colored Petri Nets as Example of High Level Nets

Modularity

Refinement

Reuse

Preparing "reducible graphs"

Colored Petri Nets, CPN

- Colored (Typed) Petri Nets (CPN) refine Petri nets:
 - Tokens are typed (colored)
 - Types are described by data structure language (e.g.,Java, ML, UML class diagrams, data dictionaries, grammars)
 - Concept of time can be added
- Full tool support
 - Fully automated code generation in Java and ML (in contrast to UML)
 - Possible to proof features about the PN
 - Net simulator allows for debugging
- Much better for safety-critical systems than UML, because proofs can be done



Annotations in CPN

- Places are annotated by
 - Token types (STRING x STRING)
 - Markings of objects and the cardinality in which they occur: 2'("Uwe", "Assmann")
- Edges are annotated by
 - Type variables which are unified by unification against the token objects (\mathbb{X},\mathbb{Y})
 - Guards
 [X == 10]
 - If-Then-Else statements if X < 20 then Y := 4 else Y := 7
 - Switch statements
 - Boolean functions that test conditions



CPN are Modular

- A subnet is called a page (module)
 - Every page has ports
 - Ports mark in- and out-going transitions/places
- Transition page: interface contains transitions (transition ports)
- Place page (state page): interface contains place (place ports)
- Net class: a named page that is a kind of "template" or "class"
 - It can be instantiated to a net "object"
- Reuse of pages and templates possible
 - Libraries of CPN "procedures" possible



Robots with Transition Pages, Coupled by Transition Ports



Robots with Place (State) Pages, Coupled by Replicated State Ports



CPN are Hierarchical

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- Places and transitions may be hierarchically refined
 - Two pointwise refinement operations:
 - Replace a transition with a transition page
 - . Replace a state with a state page
 - Refinement condition: Retain the embedding (embedding edges)

CPN can be arranged as hierarchical graphs (reducible graphs, see later)

- Large specifications possible, overview is still good
- Subnet stemming from refinements are also place or transition pages



Point-wise Refinement Example

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Pointwise refinement:

- Transition refining page: refines a transition, transition ports
- Place refining page (state refining page): refines a place, place ports





Point-wise Refinement Example

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Hyperedge refinement:

• Hyperedges and regions in PN can be refined





Modularity is Important for Scaling – Industrial Applications of CPN

- Large systems are constructed as reducible specifications
 - They have 10-100 pages, up to 1000 transitions, 100 token types
- Example: ISDN Protocol specification
 - Some page templates have more than 100 uses
 - Corresponds to millions of places and transitions in the expanded, non-hierarchical net
 - Can be done in several person weeks





3.2 Patterns in and Transformations of Petri Nets

- Petri Nets have a real advantage when parallel processes and synchronization must be modelled
 - Many concepts can be expressed as *PN patterns* or with *PN complex operators*
- Analyzability: Petri Nets can be analyzed for patterns (by pattern matching)
- Transformation: Petri Nets can be simplified by automatic transformations

Simple PN Buffering Patterns

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Reservoir Place Does not generate objects





Permanently active transaction Generates objects (Object source, Event source)





Process Sequential



Intermediate Archive



Patterns for Synchronization (Barrier)

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Coupling processes with parallel continuation





Patterns for Synchronization (n-Barrier)

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Bridges: Transitions between phases





Adding Delays in Transitions by Feedback Loops

- > Adding a **delay token**
- Behaves like a semaphore (lock – unlock critical region)







Adding Delays in Transitions by Feedback Loops

- > Adding a circular delay net
- > Behaves like a splitter







Simpler Specification with Special Operators (Transitions) in Workflow Nets

- > In languages for Workflow nets, such as
 - ARIS workflow language
 - YAWL Yet another workflow language
 - BPMN Business Process Modeling Notation
 - BPEL Business Process Execution Language
- Specific transitions have been designed (specific operators) for simpler specification



Complex Transition Operators in Workflow Nets: Join and Split Operators of YAWL

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AND-Join All ingoing places are ready (conjuctive input) AND-Split All outgoing places are filled (conjuctive output)





XOR-Join Exactly one of n ingoing places is ready (disjunctive input)

XOR-Split Exactly one of the outgoing places are filled (disjunctive output)





OR-Join At least one of n ingoing places is ready (selective input) OR-Split (IOR-Split) Some of the outgoing places are filled (selective output)





Simple YAWL example

- > OR-Booking of travel activities
- Indeterministic choice possible





Parallelism Patterns – Transitional Operators

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Joining Parallelism Synchronization Barrier AND-Join





Replication and Distribution Forking (AND-Split)





Collecting Objects From parallel processes OR-Join



Decision Indeterministically (OR-Split)







Example: Reduction Semantics of OR-Join Operator

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> Complex operators refine to special pages with multiple transition ports





Example: Reduction Semantics of XOR-Join Operator

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> XOR-Join with bound state (only 1 token can go into a place)





Parallelism Patterns – Transitional Operators (2)

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Ordering Synchronization Barrier Ordering-AND-Join







Parallelism Patterns – Transitional Operators (2)

























Patterns for Communication

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Producer Consumer with Buffer




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Producer Consumer with Buffer (size 1 message)





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Producer Consumer with Buffer (size n message)













- Binary or counting semaphores offer their lock and free operations as transitions
- > Distinguished by the capacity of the semaphore place





- Binary or counting semaphores offer their lock and free operations as transitions
- > Distinguished by the capacity of the semaphore place













Dining Philosophers (Shared Resources)





- Patterns can be used to model specific requirements
- PN can be checked for patterns by Pattern Matching (context-free Graph Rewriting)
 - Patterns can be restructured (refactorings)
 - Patterns can be composed (composition)
- PN can be simplified by graph transformation rules
- Further semantic analysis of PN: Parallel, indeterministic systems can be checked for
 - Absence of deadlocks: will the parallel system run without getting stuck?
 - Liveness: will all parts of the system work forever?
 - Fairness: will all parts of the system be loaded equally?
 - Bounded resources: will the system use limited memory, and how much? (important for embedded systems)
 - Whether **predicates hold** in certain states (model checking)





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3.3 The Application to Modelling

Petri Nets Generalize UML Behavioral Diagrams

Activity Diagrams

- Activity Diagrams are similar to Marked Graphs, but not formally grounded
 - Without markings
 - No liveness analysis
 - No resource consumption analysis with boundness
 - No correspondence to UML-Statechart
- Difficult to prove something about activity diagrams and difficult to generate parallel code

Data-flow diagrams

- DFD are special form of activity diagrams
 - Non-shared-memory DFD correspond to Marked Graphs

Statecharts

- Finite automata are restricted form of Petri nets
- Hierarchical structuring in Statecharts is available in High-Level Petri Nets (e.g., CPN)



Petri Nets Generalize UML Sequence Diagrams

- The life lines of a sequence diagram can be grouped into state such that a PN results
- > All of a sudden, liveness conditions can be studied
 - Is there a deadlock in the sequence diagram?
 - Are objects treated fair?





Petri Nets Generalize UML Sequence Diagrams

- The life lines of a sequence diagram can be grouped into state such that a PN results
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A Simple Modelling Process for Safety-Critical Software with CPN

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Elaboration:

- 1. Identify active and passive parts of the system
 - Active become transitions, passive to places
- 2. Find the relations between places and transitions
- 3. How should the tokens look like: boolean? Integers? Structured data?
 - Active become transitions, passive to places
- Restructure: Group out subnets to separate "pages"
- Refactor: Simplify by reduction rules
- Verify: Analyse the specification on liveness, boundedness, reachability graphs, fairness. Use a model checker to verify the CPN
- Transform Representation: Produce views as statecharts, sequence, collaboration, and activity diagrams.



How to Solve the Reactor Software Problem?

- Specify the reactor core with UML and CPN
 - Map the static parts to the net
 - Map the flow of things to tokens
 - Map the state chances to token flow
 - Think about synchronizations
 - Specify in PN views
- Verify it with a model checker
 - Let a prototype be generated
 - Test it
 - Freeze the assembler
- Verify the assembler, because you should not trust the CPN tool nor the compiler
 - Any certification agency in the world will require a proof of the assembler!
- However, this is much simpler than programming reactors by hand...



The Gloomy Future of PN

- PN will become the major tool in a future CASE tool or IDEs
 - Different views on the PN: state chart view, sequence view, activity view, collaboration view!
- Many isolated tools for PN exist, and the world waits for a full integration into UML
- CPN will be applied in scenarios where parallelism is required
 - Architectural languages
 - Web service langauges (BPEL, BPMN, ...)
 - Workflow languages
 - Coordination languages



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> Thanks to Björn Svensson for help to summarize [Murata] in slides

