

# 21. Technical Space TreeWare

## Simplification and Attribute Analysis on Trees

### How to Simplify and Interpret Programs and Models

Prof. Dr. rer. nat. Uwe Aßmann

Institut für Software- und  
Multimediatechnik

Lehrstuhl Softwaretechnologie

Fakultät für Informatik

Technische Universität  
Dresden

[http://st.inf.tu-dresden.de/  
teaching/most](http://st.inf.tu-dresden.de/teaching/most)

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- 1) Tree Simplification and Tree Rewriting
- 2) Analysis on Trees
  - 1) Metric Interpretation
  - 2) Attribute Analysis
- 3) Attributed Trees and Attributed Grammars for Interpreters on Trees



DRESDEN  
concept  
Exzellenz aus  
Wissenschaft  
und Kultur

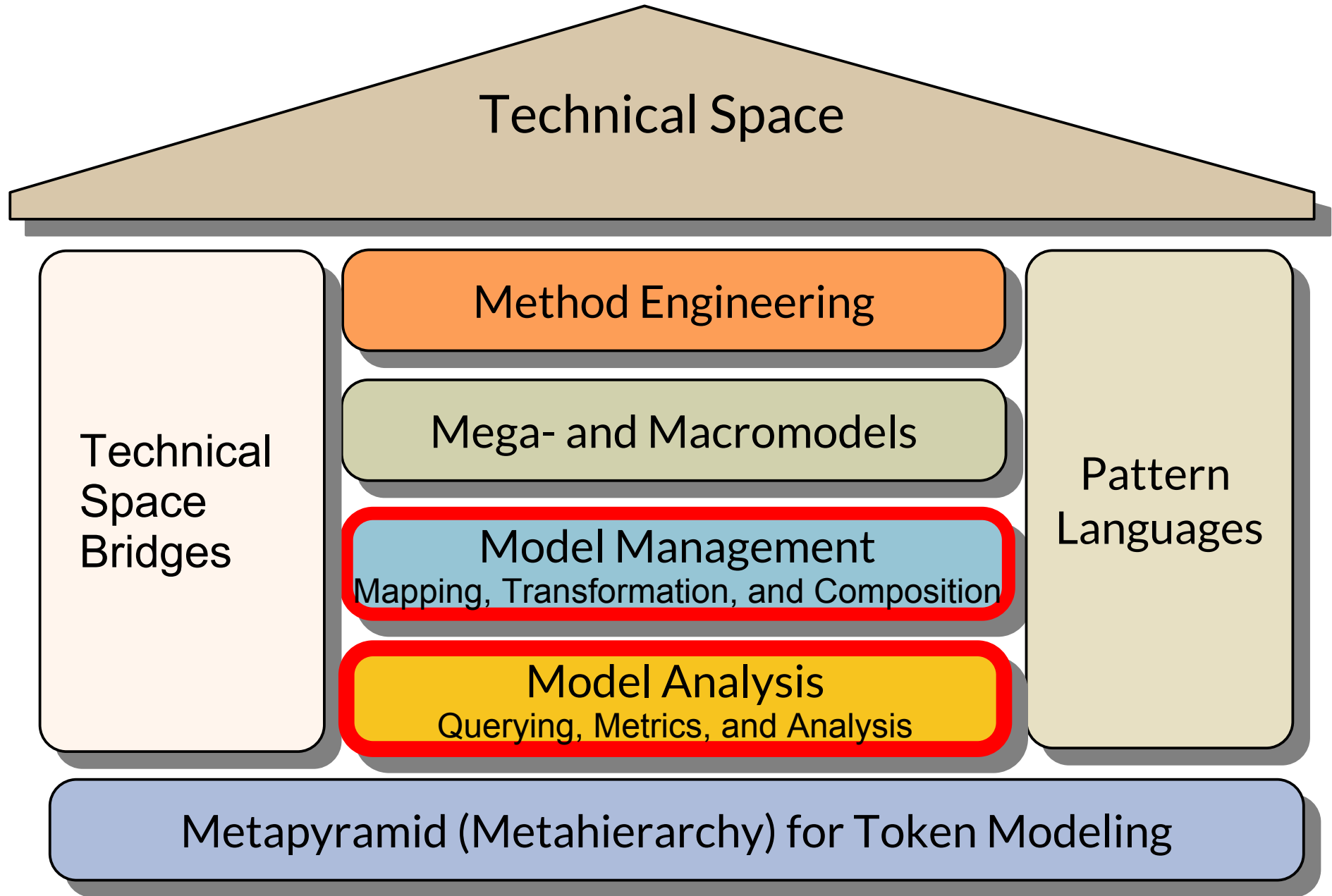
# Obligatory Literature

- ▶ List of analysis tools
  - [http://en.wikipedia.org/wiki/List\\_of\\_tools\\_for\\_static\\_code\\_analysis](http://en.wikipedia.org/wiki/List_of_tools_for_static_code_analysis)
- ▶ Paakki, Jukka. 1995. „Attribute grammar paradigms—a high-level methodology in language implementation“. ACM Comput. Surv. 27 (2) (Juni): 196–255.
- ▶ [KSV09] Lennart C. L. Kats, Anthony M. Sloane, Eelco Visser. Decorated Attribute Grammars. Attribute Evaluation Meets Strategic Programming (Extended Technical Report). Report TUD-SERG-2008-038a, Delft University
- ▶ [SKV09] Anthony M. Sloane, Lennart C. L. Kats, Eelco Visser. A Pure Object-Oriented Embedding of Attribute Grammars. Report TUD-SERG-2009-004, Delft University
- ▶ [LLL] Rüdiger Lincke, Jonas Lundberg and Welf Löwe. Comparing Software Metrics Tools

# Other Literature on Attribute(d) Grammars

- ▶ Knuth, D. E. 1968. „Semantics of context-free languages“. Theory of Computing Systems 2 (2): 127–145.
- ▶ Hedin, Görel. 2000. „Reference Attributed Grammars“. Informatica (Slovenia) 24 (3): 301–317.
- ▶ Boyland, John T. 2005. „Remote attribute grammars“. Journal of the ACM 52 (4) (Juli): 627–687.
- ▶ Bürger, Christoff, Sven Karol, Christian Wende, und Uwe Aßmann. 2021. „Reference Attribute Grammars for Metamodel Semantics“. In Software Language Engineering, LNCS 6563:22–41.
- ▶ Examples on: [www.jastemf.org](http://www.jastemf.org)

# Q10: The House of a Technical Space



# Glossary for Automated Rewriting on Strings, Terms and Graphs

- ▶ **Rewrite rule:** rule (left, right hand side) to match left-hand side in the graph and to transform it to the right-hand side
- ▶ **Rewrite system (RS):** set of graph rewrite rules
- ▶ **Start data (axiom):** input data to rewriting process
- ▶ **Rewrite problem:** a rewrite system applied to a start data
- ▶ **Manipulated data (host data):** data which is rewritten in rewrite problem
- ▶ **Redex (reducible expression):** application place of a rule in the manipulated data
- ▶ **Rule mapping:** the mapping of a rule to a redex
- ▶ **Normal form:** result data of rewriting; manipulated data without further redex
- ▶ **Derivation:** a sequence of rewrite steps on the manipulated graph, starting from the start data and ending in the normal form
- ▶ **Unique normal form:** unique result of a rewrite system, applied to one start data
- ▶ **Deterministic RS:** rewrite system with one normal form
- ▶ **Terminating RS:** rewrite system that stops after finite number of rewrites
- ▶ **Confluent RS:** two derivations always can be commuted, resp. joined together to one result
- ▶ **Strong confluent RS:** all pairs of rewrite steps can be commuted
- ▶ **Convergent RS:** terminating deterministic rewrite system that always yields unique results (equivalent to terminating and confluent)

## 21.1 Simplification - Rewritings with the Stratego Term Rewriting System



# Term and Tree Rewrite Systems (Termersetzungssysteme, TRS)

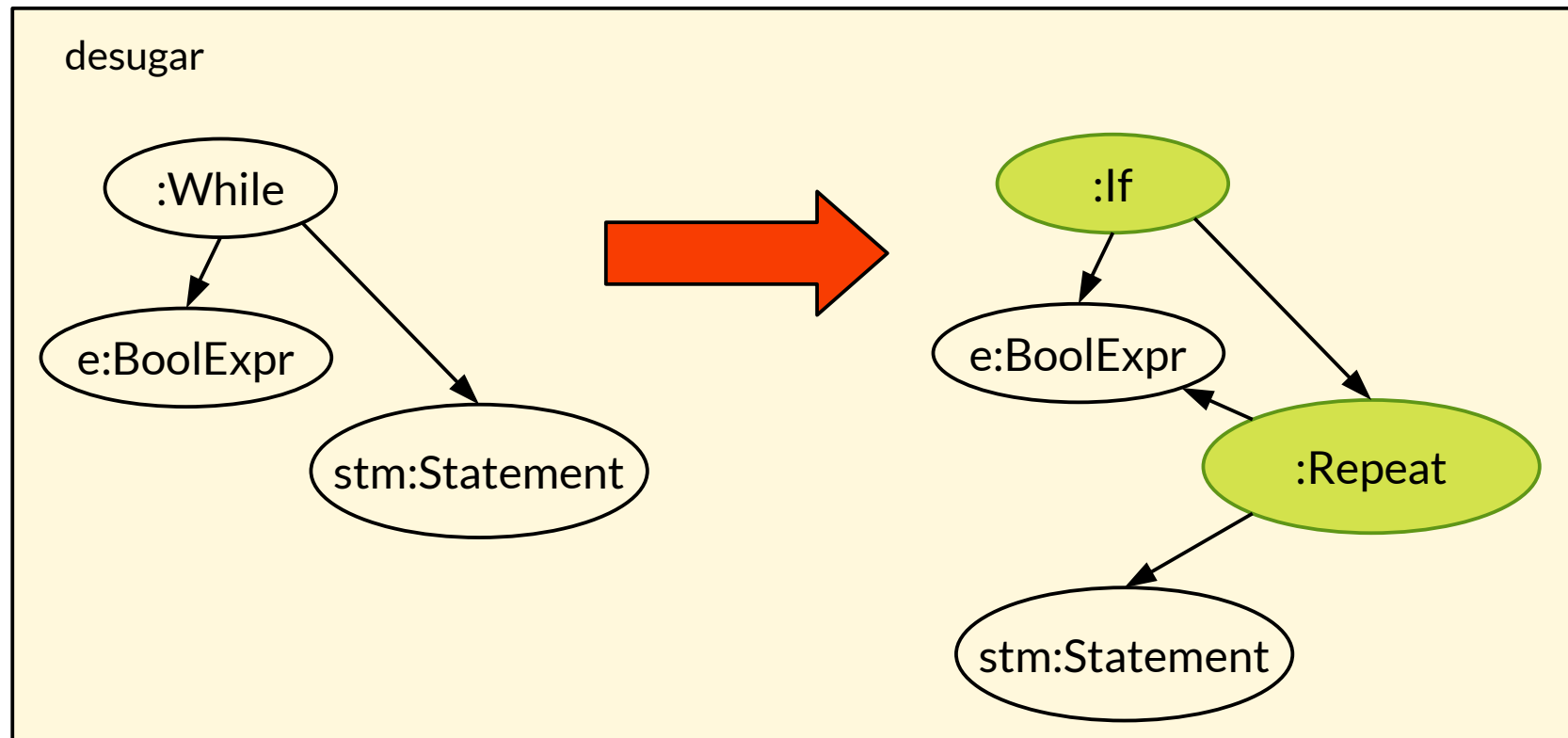
- ▶ Rewrite Systems enable the specification of **transformative semantics (reductive semantics)**
  - They reduce a data structure to a normal form, i.e., “give it a semantics”
  - They apply rewrite rules until a fixpoint
- ▶ **Term rewrite systems (Termersetzungssysteme)** transform tree- or term data structures
  - Can be used to rewrite an (abstract) syntax tree (AST)
  - Based on RTG
  - If pattern is a unordered tree, we speak of **tree rewriting**
  - If pattern is a term (ordered tree), we speak of **term rewriting**
- ▶ Use:
  - **Identification** of tree patterns (pattern matching)
  - **Simplifications** such as peephole optimization, constant folding
  - **Normalisations**, such as expanding abbreviations
  - **Inlining** and **outlining** of functions

# Stratego Term Rewrite System

- ▶ Syntax of a Stratego rewrite rule is based on RTG patterns

Name : RTG-Pattern „->“ Pattern

```
// Example: lowering all While statements to If statement with Repeats  
desugar : While(e, stm) -> If(e, Repeat(stm, e))
```

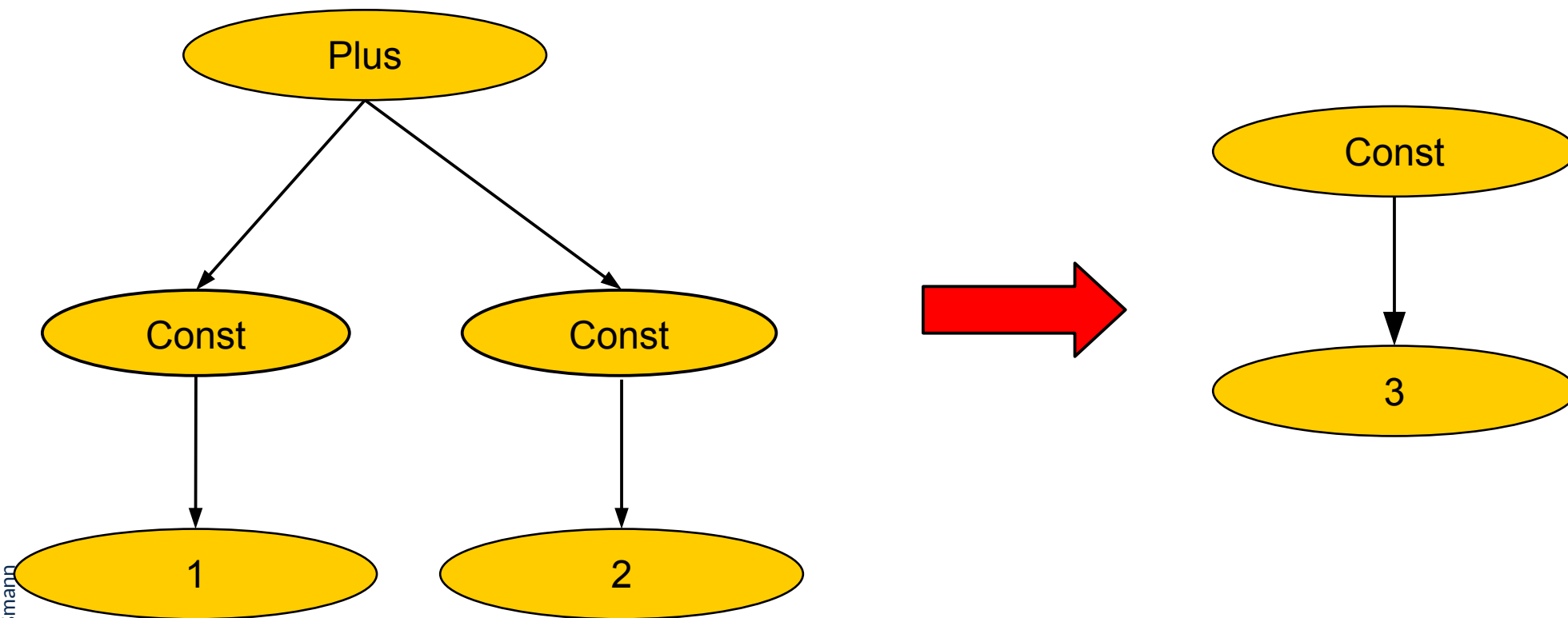




# Constant Folding as Subtractive TRS

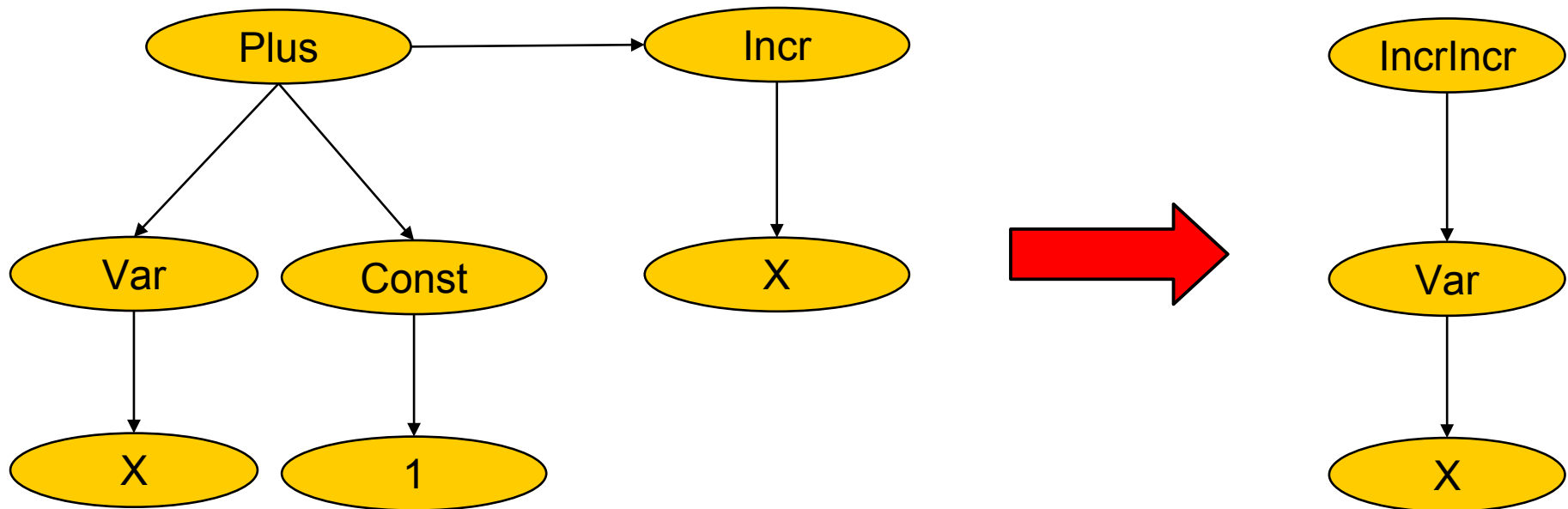
// Example: a special case of constant folding as RTG pattern rewriting

foldPlus : Plus(Const(1), Const(2)) -> Const(3)



# Peephole Optimization as Subtractive TRS

// Example: a special case of peephole optimization  
peepPlusIncr : next(Plus(Var(X),Const(1)), Incr(X)) ->  
IncrIncr(Var(X))



# An Constant Folder Programmed in Stratego

- ▶ Constant folding in the TIL language
- ▶ <http://hydra.nixos.org/build/23332578/download/1/manual/chunk-chapter/examples.html>

```
// constant folding in Stratego
module til-eval
imports TIL
rules
compare(s) = if s then !True() else !False() end
EvalAdd:  Add(Int(i), Int(j)) -> Int(<addS>(i,j))
EvalAdd:  Add(String(i), String(j)) -> String(<conc-strings>(i,j))
EvalSub:  Sub(Int(i), Int(j)) -> Int(<subtS>(i,j))
EvalMul:  Mul(Int(i), Int(j)) -> Int(<mulS>(i,j))
EvalDiv:  Div(Int(i), Int(j)) -> Int(<divS>(i,j))
EvalMod:  Mod(Int(i), Int(j)) -> Int(<modS>(i,j))
EvalLt:   Lt(Int(i), Int(j)) -> <compare(ltS)>(i,j)
EvalGt:   Gt(Int(i), Int(j)) -> <compare(gtS)>(i,j)
EvalLeq:  Leq(Int(i), Int(j)) -> <compare(leqS)>(i,j)
EvalGeq:  Geq(Int(i), Int(j)) -> <compare(geqS)>(i,j)
EvalEqu:  Equ(Int(i), Int(j)) -> <compare(eq)>(i,j)
EvalOr:   Or(True(), e) -> True()
EvalOr:   Or(False(), e) -> e
EvalAnd:  And(True(), e) -> e
EvalAnd:  And(False(), e) -> False()
AddZero:  Add(e, Int("0")) -> e
AddZero:  Add(Int("0"), e) -> e
MulOne:   Mul(e, Int("1")) -> e
MulOne:   Mul(Int("1"), e) -> e
EvalS2I:  FunCall("string2int", [String(x)]) -> Int(x)
          where <string-to-int> x

EvalI2S:  FunCall("int2string", [Int(i)]) -> String(i)
EvalIf:   IfElse(False(), st1*, st2*) -> Block(st2*)
EvalIf:   IfElse(True(), st1*, st2*) -> Block(st1*)
EvalWhile: While(False(), st*) -> Block([])
```

# Stratego System

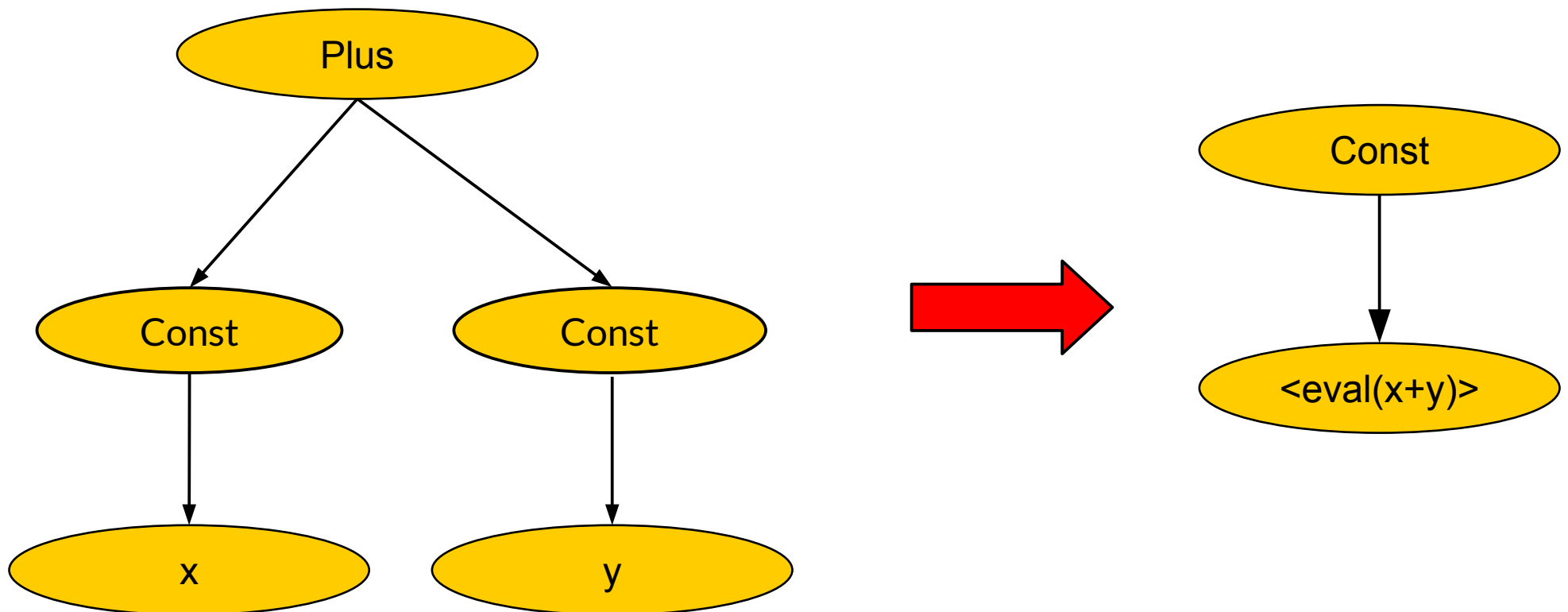
- ▶ TRS compiled to C
  - Terms represented with the C-based Aterm library
- ▶ TRS compiled to Java
  - Rewriting the Java-based syntax trees of Eclipse JDT

# Rewriting Strategies

- **Free (chaotic) rewriting:** all rules are applied until a fixpoint, the point of no change
- **Confluent rewriting:** when free rewriting ends up always with the same result (same normalform), the rewriting is confluent
- **Strategies** are second order rules that can steer the application of normal, first-order rules:
  - Top-down  $\text{topdown}(r)$
  - Bottom-up  $\text{bottumup}(\text{try}(r))$
  - Left-to-right depth-first / breadth-first
  - Right-to-left depth-first / breadth-first
  - Try a rule  $\text{try}(r) = r \lt+ \text{id}$
- Strategies are important for non-confluent rewriting problems
- Ex.: Das alternierende Suchen und Löschen von gefundenen Informationen.

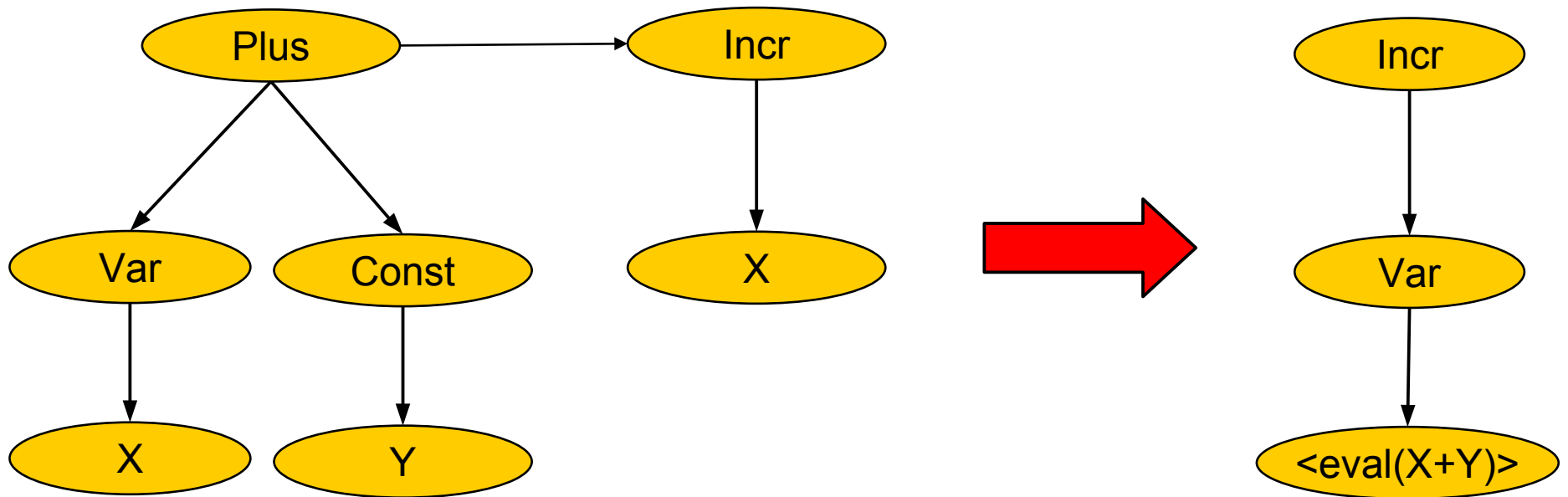
# Constant Folding with Strategic Rewriting

```
// Evaluation goes bottom-up of a (possibly big) term  
foldPlusOperations: bottomup(Plus(Const(x), Const(y)) ->  
Const(<eval(x+y)>))
```



# Peephole Optimization with Topdown Rewriting

```
// Example: a more general case of peephole optimization  
peepConstants:  
topdown(next(Plus(Var(X),Const(Y)), Incr(X))) -> Incr(Var(<eval(X+Y-1)>))
```



## 21.1.2 The TXL Tool

- ▶ <External slide set from TXL distribution: TXLintro.pdf>
  - <http://www.txl.ca/docs/TXLintro.pdf>
- ▶ Exercise: Download the TXL distribution (10.7) from
  - <http://www.txl.ca/>
- ▶ Investigate the TXL Pascal Grammar.
  - [examples/analysis/tracing/Txl/Pascal.Grm](#)
- ▶ Then look at the ptract.txl specification, which adds tracing statements to a Pascal program:
  - [examples/analysis/tracing/Txl/ptrace.Txl](#)
- ▶ Try to run it and see the result.

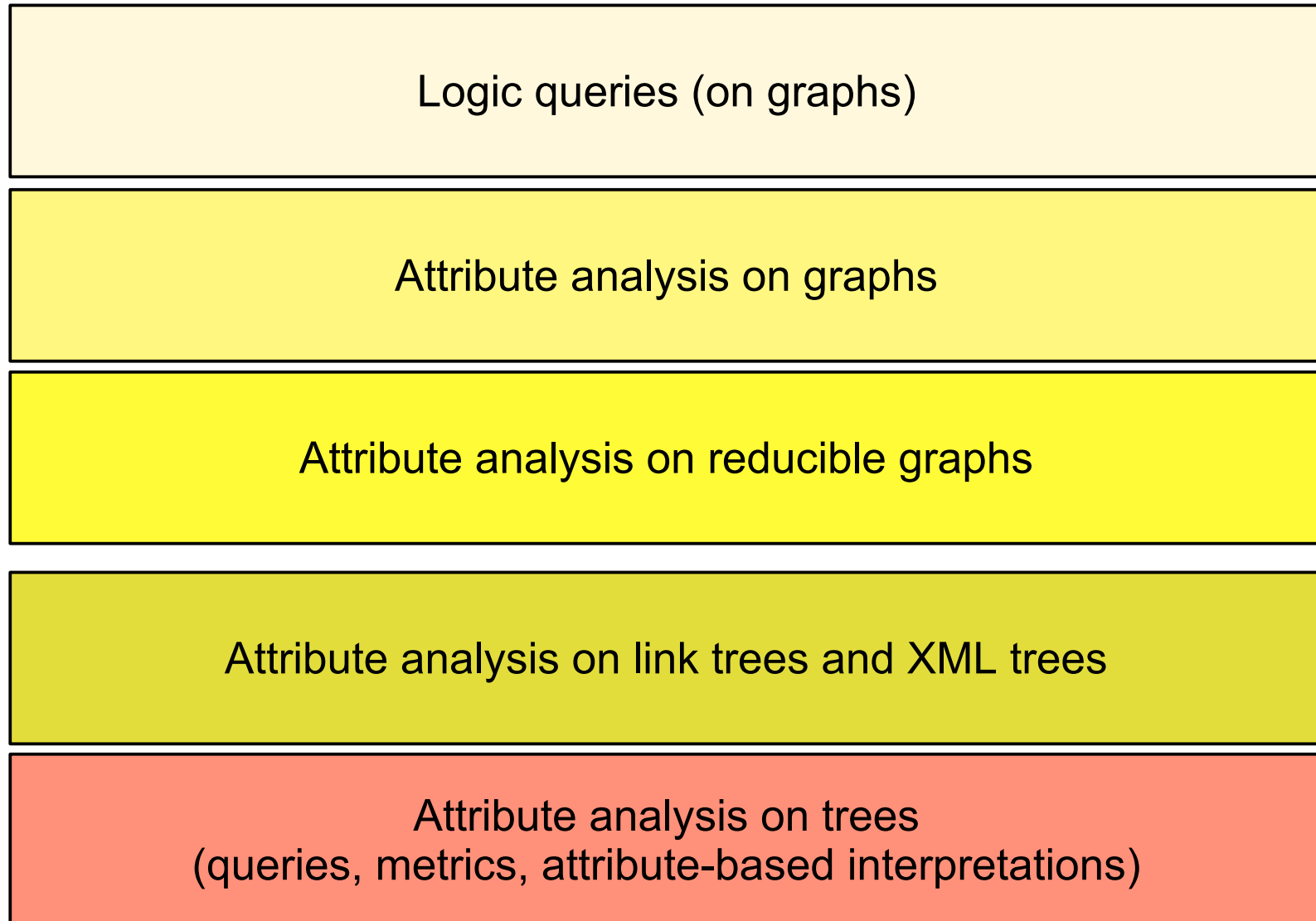


## 21.2 Analysis in an MDSD Tool

- ▶ **Analysis** in an MDSD tool requires queries, metrics, and deep analysis



# The Hierarchy of Analyses



## 21.2.1 Metric Analysis in an MDSD Tool

- ▶ **Analysis** in an MDSD tool requires queries, metrics, and deep analysis
- ▶ Queries are done in a query language, see later



- ▶ **Coupling metrics** measure the coupling of two packages, classes or modules
  - CBO: “Coupling between object classes” counts links to other classes
  - RFC: “Response for a class” counts the number of methods called in response to a message to an object
- ▶ **Cohesion metrics** measure the cohesion of one package, class, or module
  - LCOM “Lack of cohesion of methods” in a unit
- ▶ **Inheritance metrics**
  - DIT: “Depth of inheritance tree”
  - BIT: “Breadth of inheritance tree”
- ▶ **Size and complexity metrics**
  - LOC: “lines of code” - quite weak metrics
  - LOP: “Lines of procedures” - how long is a procedure
  - EXC: “expression count” in a method
  - NOM “number of methods” in a class
  - WMC “weighted methods per class” with cyclomatic complexity

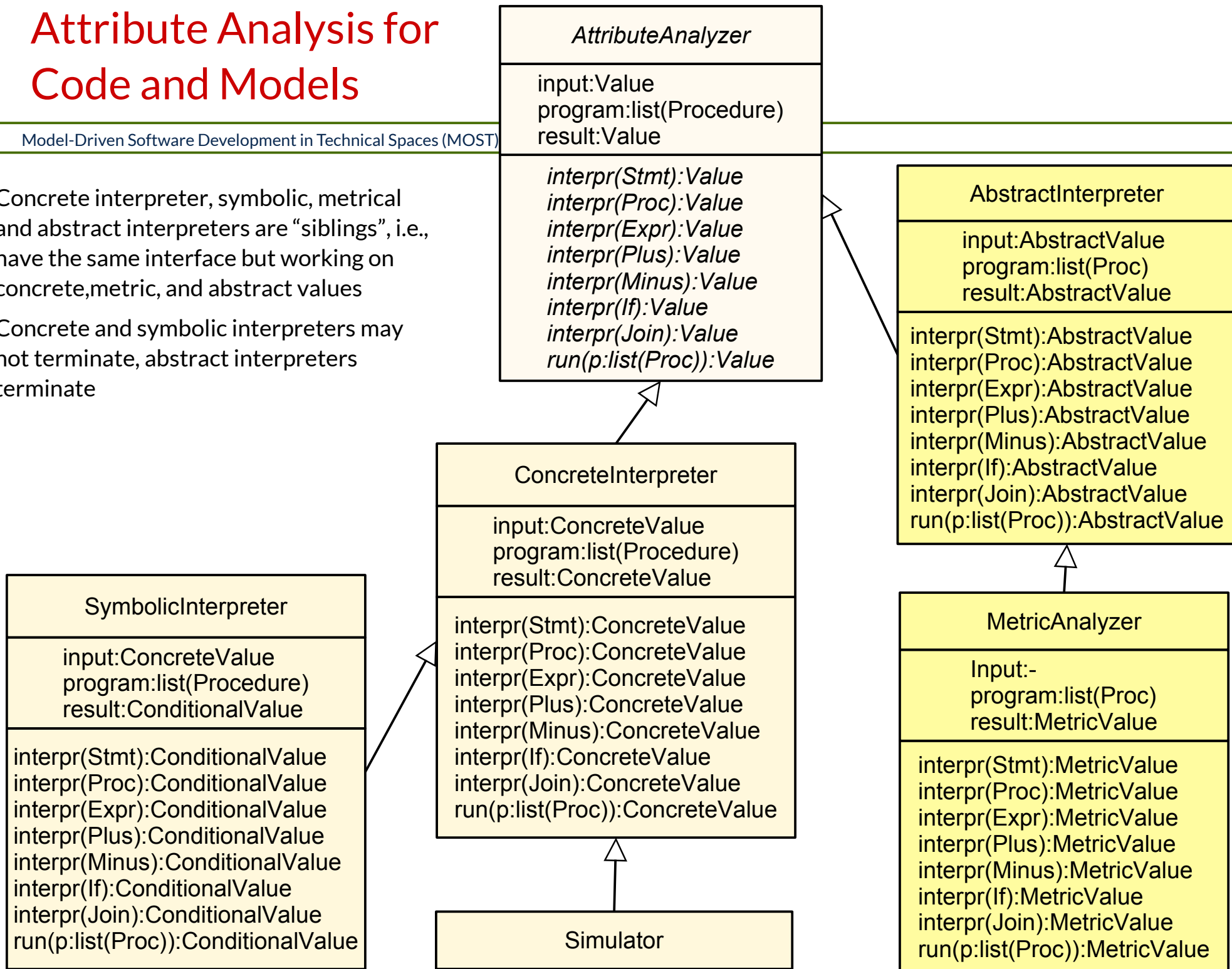
## 21.2.2 Attribute-based Analysis and Interpretation

- ▶ In **attribute-based analysis**, the code or the model stays invariant, but **attributes** are evaluated on the code, based on *stencil functions (transfer functions, attribution functions)*
- ▶ A **symbolic interpreter** executes the program, but builds up a lookup table, under which conditions which values are produced. The table may be infinitely large.



# Attribute Analysis for Code and Models

- ▶ Concrete interpreter, symbolic, metrical and abstract interpreters are “siblings”, i.e., have the same interface but working on concrete, metric, and abstract values
- ▶ Concrete and symbolic interpreters may not terminate, abstract interpreters terminate



## 21.3 Attribute(d) Grammars for Interpretation, Simulation, Metric, and Abstract Interpretation



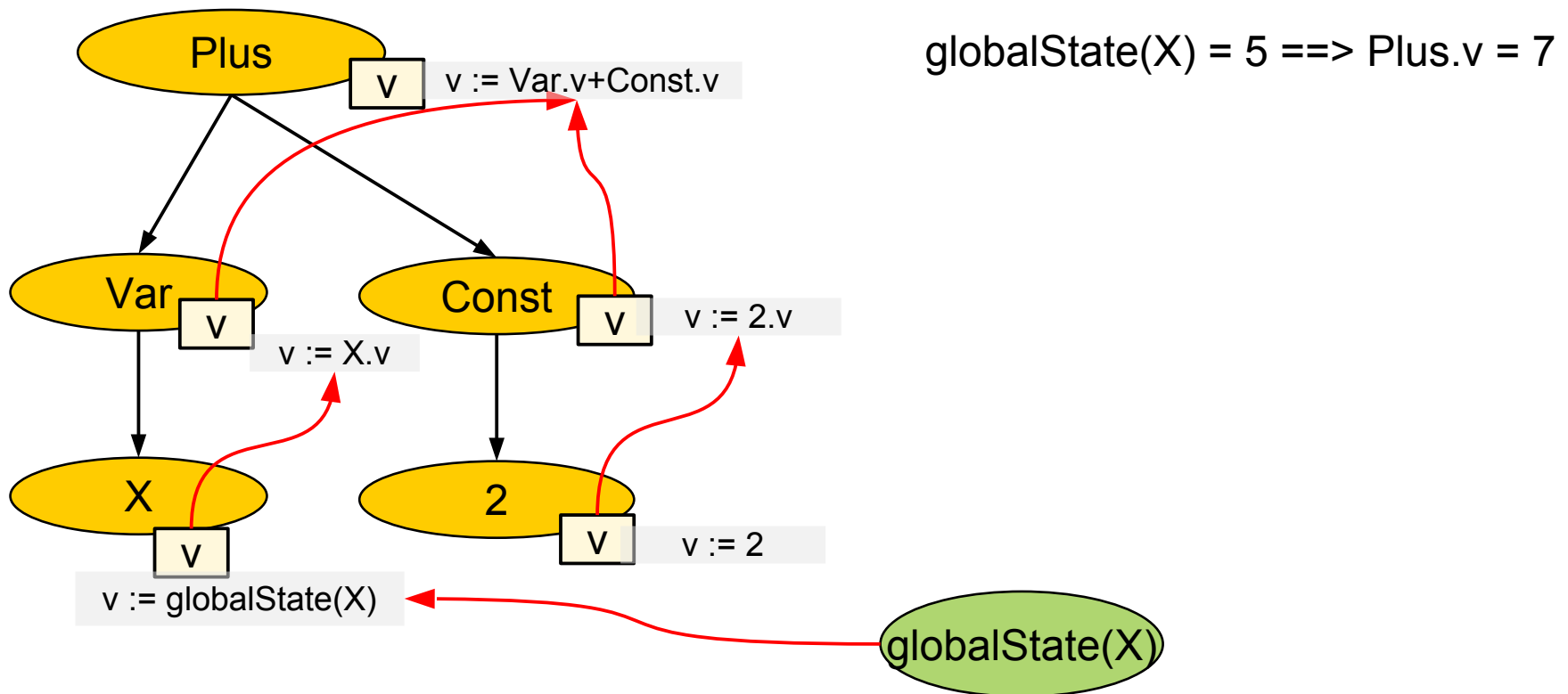
# Attribute(d) Grammars (AG) for Interpreters on Piecelists and Trees

- ▶ An **attributed tree** is a tree with attributes (record tree).
- ▶ An **attribution function (stencil)** reads and stores its results from and into the tree.
- ▶ An **attribute(d) grammar** is a regular tree grammar (RTG) in which all nonterminals are adorned with attributes, and every rule contains a set of **attribution functions (attribute equations, stencil functions)**
  - The stencil functions write their results into the tree (**attribution**)
  - AG are declarative and partition the stencil function space with their tree nodes (tree-node-specific functions)
- ▶ Instead of rewriting a tree for interpretation, AG compute their results with functions over attributes
  - **Data-Driven Programming:** the data structure of the tree is in primary focus, the attribution follows
- ▶ Attributed grammars describe calculations on trees (part lists, piece lists, Stücklisten), e.g., for
  - Metrics on piece lists in information systems
  - Concrete interpretation, metric, and abstract interpretation on syntax trees of programs
  - Computation of visualization attributes on trees



# Stencils

- ▶ AG are **stencil computation systems**, because they keep the trees invariant, but compute tree attributes with stencil functions over them
  - The tree grammar computes the part list (phase 1)
  - The stencil functions compute the attributes (phase 2)
  - The rewrites rebuilt the part list (optional phase 3)



# Interpretation of Programs with Attribute(d) Grammars (AG)

- ▶ An **attribute(d) grammar** describes an interpreter on a syntax tree (a hierarchical program representation)
  - The (dynamic) syntax tree is described by a (finite) Regular Tree Grammar
  - The nodes of the program in the syntax tree are augmented with values, **attributes**.
    - The resulting data structure is called **attributed syntax tree (AST)**
    - (Graph representations are not possible in pure AGs)
  - There is a set of **attribution functions (attribution rules, attribute equations)** which define **interpretation functions** on all nodes of the syntax tree
  - Usually, the rules are interpreted with recursion along the attributed syntax tree
- ▶ Because the underlying program representation is hierarchic, often
  - AG-based interpreters can be proven to terminate
  - can be compiled to code, instead of interpreted (pretty fast)

**AG-based concrete interpreters can analyze syntax trees by concrete interpretation evaluating their attribution functions**

# The Pattern-Major Form of AG (Node-Based Form, Window Form)

- ▶ In the **pattern-major form (window form)** of an AG, the tree node patterns of the RTG used to describe the tree form the *windows* (the major groups of attribute definitions)
- ▶ Attribution Functions are written in a functional language. They take node attributes as parameters and results.
- ▶ A **stencil** is an assignment of an attribute by a attribution function.
  - In one window, many stencils may appear.

```
Interpretation evalArithmeticExpr(Tree → Tree)
  in pattern-major form {
  Attribute definitions of Root(st) {
    this.result := st.result;
    <println(„Result is %S“, this.result)>
  }
  Attribute definitions of Plus(st1,st2) {
    this.result := <st1.result + st2.result>
  }
  Attribute definitions of Minus(st1,st2) {
    this.result := <st1.result - st2.result>
  }
  Attribute definitions of Mult(st1,st2) {
    this.result := <st1.result * st2.result>
  }
  Attribute definitions of Div(st1,st2) {
    this.result := if (st2 == 0) then {
      <println(„Error, div by zero“)>;
      -999 }
    else <st1.result / st2.result>
  }
  Attribute definitions of Leaf(value:Integer) {
    this.result := value
  }
}
```

# Ex.: Global Minima

- ▶ Transforming a tree to a new tree with leaf nodes carrying the global minimum of all leaf nodes

```
Transformation repmin(Tree → Tree) in pattern-major form
{
  Attribute definitions of Root(st) {
    st.global-min := st.min
    this.min := st.min
    this.replace := Root(st.replace)
  }
  Attribute definitions of Pair(st1,st2) {
    st1.global-min := this.global-min
    st2.global-min := this.global-min
    this.min := <min(st1.min,st2.min)>
    this.replace := Pair(st1.replace,st2.replace)
  }
  Attribute definitions of Leaf(value:Integer) {
    this.min := value
    this.replace := Leaf(this.global-min)
  }
}
```

# Attribute-Major Format

- ▶ In **attribute-major format**, attribution functions are *sorted along attributes*, while pattern matching is used inside the definition of the attribution function

Transformation `repmin(Tree → Tree)` in **attribute-major form** {

Attribute definitions for `min`: {

`Root(st)` → `this.min := st.min`

`Pair(st1, st2)` → `this.min := <min(st1.min, st2.min)>`

`Leaf(value: Integer)` → `this.min := value`

}

Attribute definitions for `global-min`: {

`Root(st)` → `st.global-min := st.min`

`Pair(st1, st2)` → `st1.global-min := this.global-min`

`st2.global-min := this.global-min`

`Leaf(value: Integer)` → `st.global-min := st.min`

}

Attribute definitions for `replace` { // tree-valued attribute

`Root(st)` → `this.replace := Root(st.replace)`

`Pair(st1, st2)` → `this.replace := Pair(st1.replace, st2.replace)`

`Leaf(value: Integer)` → `this.replace := Leaf(this.global-min)`

}

}

# Function-Major Format (very similar to functional programming)

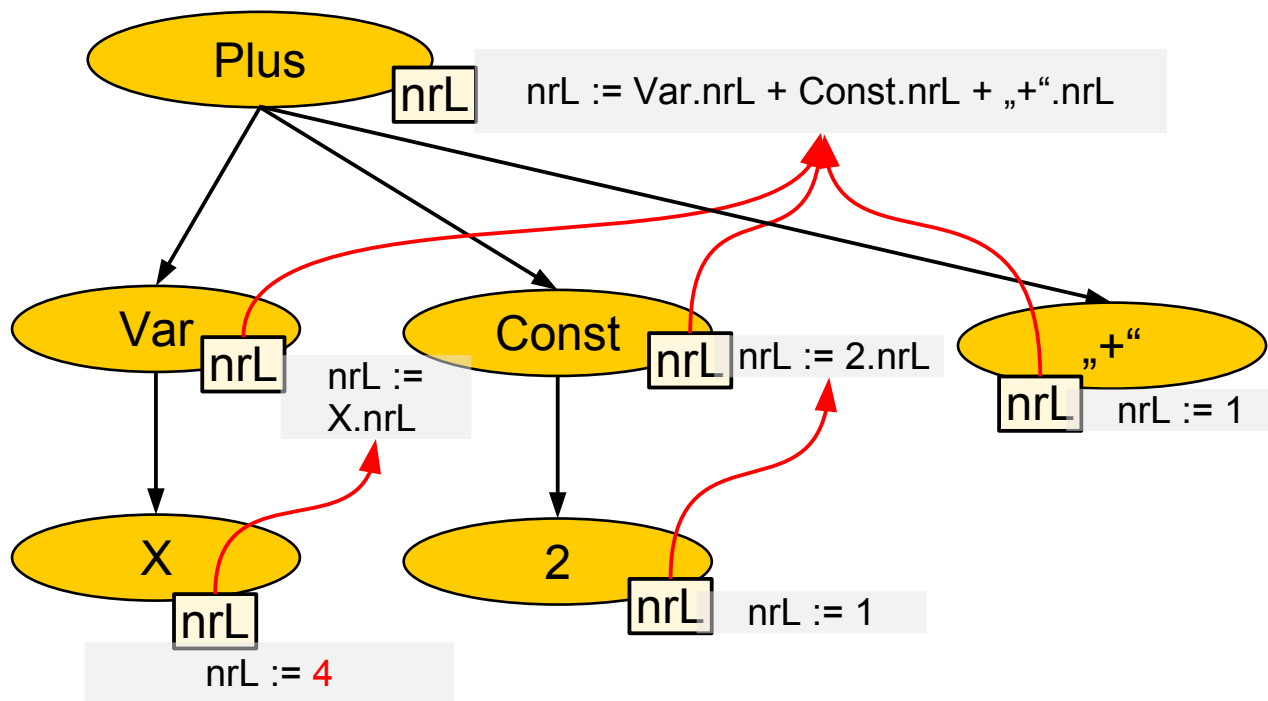
- ▶ Attribution functions are sorted *along functions* of a functional program, in which pattern matching is used inside the definition of the attribution function

```
Transformation repmin(Tree → Tree) in function-major form {  
  function compute_min: {  
    Root(st)          → this.min := compute_min(st.min)  
    Pair(st1,st2)     → this.min := compute_min(st1.min,st2.min)  
    Leaf(value:Integer) → this.min := id(value)  
  }  
  function compute_global-min: {  
    Root(st)          → st.global-min := compute_min(st.min)  
    Pair(st1,st2)     → st1.global-min := compute_min(this.global-min)  
                      st2.global-min := compute_min(this.global-min)  
    Leaf(value:Integer) → st.global-min := compute_min(st.min)  
  }  
  function compute_replace { // tree-valued attribute  
    Root(st)          → this.replace := compute_replace(st.replace)  
    Pair(st1,st2)     → this.replace := compute_replace(st1.replace,st2.replace)  
    Leaf(value:Integer) → this.replace := compute_replace(this.global-min)  
  }  
}
```

# Attributed Grammars (AG) can Specify Metric Analyzers

- ▶ An attributed grammar can describe a **metric analyzer**, if the values are from a domain of a software metrics
- ▶ Then, the set of attribution rules (attribute equations) define a software metrics interpretation functions on the syntax tree

```
// simple expression  
// distributed over  
// several lines  
X  
+  
2
```



**AG-based abstract interpreters can analyze syntax trees by metric interpretation**

# Attributed Grammars (AG) Can Specify Abstract Interpreters

- ▶ An attributed grammar can describe an **abstract interpreter**, if the values are from an abstract domain (a system of equivalence classes)
  - e.g., from a set of types, a type system, interval ranges, etc.
  - Then, the set of attribution rules (attribute equations) define abstract interpretation functions computing on equivalence classes
- ▶ Example: **Type analysis and checking**
  - The analysis of expressions on their types (int, real, char, string, etc) and the check whether their types are compatible is an abstract interpretation
  - Finitely many types ((int, real, char, string, user types)
  - Inclusion and compatibility rules for types
    - Char < int < real
    - Range < int
    - Person < Object

**AG-based abstract interpreters can analyze syntax trees by abstract interpretation**



# The End

- ▶ Explain the differences of a concrete interpreter, a metric analyzer, and an abstract interpreter
- ▶ What are the differences of an abstract interpreter and an attribute grammar?
- ▶ Why is a reference attribute grammar more expressive than a pure AG?
- ▶ What happens at a control-flow join during an interpretation?
- ▶ Why is *metric interpretation* important?
- ▶ Explain how RTG and AG are related
- ▶ Explain the difference of pattern-major and attribute-major form
- ▶ What is the difference of a functional program and an AG?
- ▶ Why is an abstract interpreter a functional program?