41. Deep Graph Model Analysis and Macromodels: Model and Program Analysis with (Recursive) Graph Reachability

How Context-Sensitive Constraints can be Checked in a Model

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1) Graph Reachability as Deep Analysis
1) EARS
   1) Regular graph reachability and Slicing
2) Graph slicing
3) Value-flow analysis
   1) Context-free graph reachability
4) More on the Graph-Logic Isomorphism
   1) Implementation in Tools
5) Model Mappings in Megamodels

Literature

- GrGen web site http://www.info.uni-karlsruhe.de/software/grgen/
- GrGen User Manual http://www.info.uni-karlsruhe.de/software/grgen/GrGenNET-Manual.pdf
- [Aßmann00] Uwe Aßmann. Graph rewrite systems for program optimization. ACM Transactions on Programming Languages and Systems (TOPLAS), 22(4):583-637, June 2000.
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Literature on the Graph-Logic-Isomorphism

- B. Courcelle. Graphs as relational structures: An algebraic and logical approach. In H. Ehrig, H.-J. Kreowski, and G. Rozenberg, editors, 4th International Workshop On Graph Grammars and Their Application to Computer Science, volume 532 of Lecture Notes in Computer Science, pages 238-252. Springer, March 1990.
- B. Courcelle. The logical expression of graph properties (abstract). In H. Ehrig, H.-J. Kreowski, and G. Rozenberg, editors, 4th International Workshop On Graph Grammars and Their Application to Computer Science, volume 532 of Lecture Notes in Computer Science, pages 38-40. Springer, March 1990.
- B. Courcelle. Graph rewriting: An algebraic and logic approach. In Jan van Leeuwen, editor, Handbook of Theoretical Computer Science, pages 193- 242, Amsterdam, 1990. Elsevier Science Publishers.

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- Uwe Aßmann. OPTIMIX, A Tool for Rewriting and Optimizing Programs. In Graph Grammar Handbook, Vol. II. Chapman-Hall, 1999.
- K. Lano. Catalogue of Model Transformations
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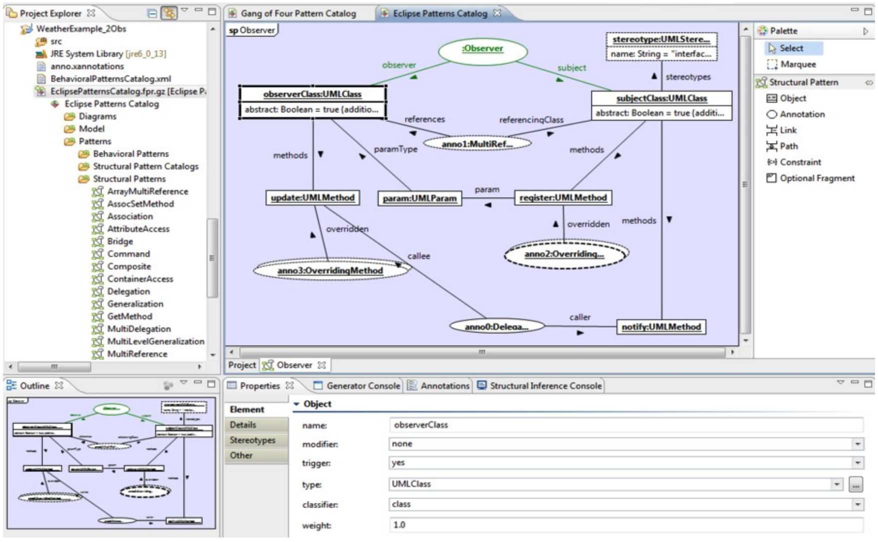
41.1. Introduction to Diagrammatic Storyboard Rule Notation for Graph Rewriting

Originally introduced by Fujaba www.fujaba.de (tool now unsupported)

Fujaba

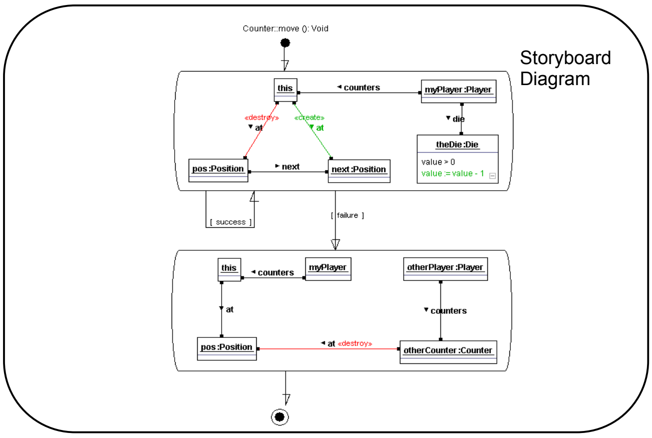
- Fujaba is a MetaCASE-tool based on GRS with home-grown metalanguage and metamodel
- Basic technology: graph pattern matching and rewriting



http://www.fujaba.de/typo3temp/pics/604c5c6c9e.png

Fujaba Storyboard Diagrams for Adding and Removing Graph Fragments

- Storyboards are activity diagrams in which activities are GRS (graph notation with colors)
- Green color: adding model fragments; Red color: deleting them
- Pool starts at node this and reaches into the object net
- GRS can be embedded into Petri Nets, DFG and other BSL

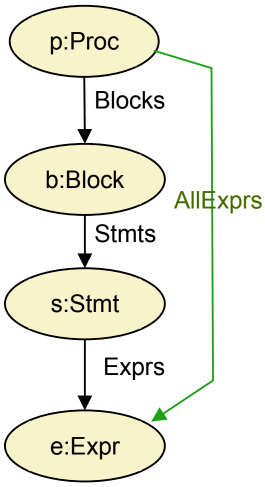


41.1 Using EARS (Binary Edge Addition)for Deep Analysis of Models and Mappings of Models and Code
- and as the Bridge to Graph Rewriting

- Graph reachability engines are analysis tools answering questions about the deeper structure of models and programs
- EARS can be employed for regular graph reachability, context-free graph reachability, slicing, data-flow analysis
- And traceability for inter-model relationships

EARS for Model Mapping

- **Edge addition rewrite systems (EARS)** compute direct binary relations for remotely reachable parts of a graph and a model
  - They **abbreviate long** paths in models
- EARS can be used for reachability of elements in models and model mapping:
  - Transitive closure
  - Regular path reachability
  - Context-free path reachability
- EARS form the bridge to graph rewriting and graph-rewriting based model transformations
- They correspond to binary Datalog



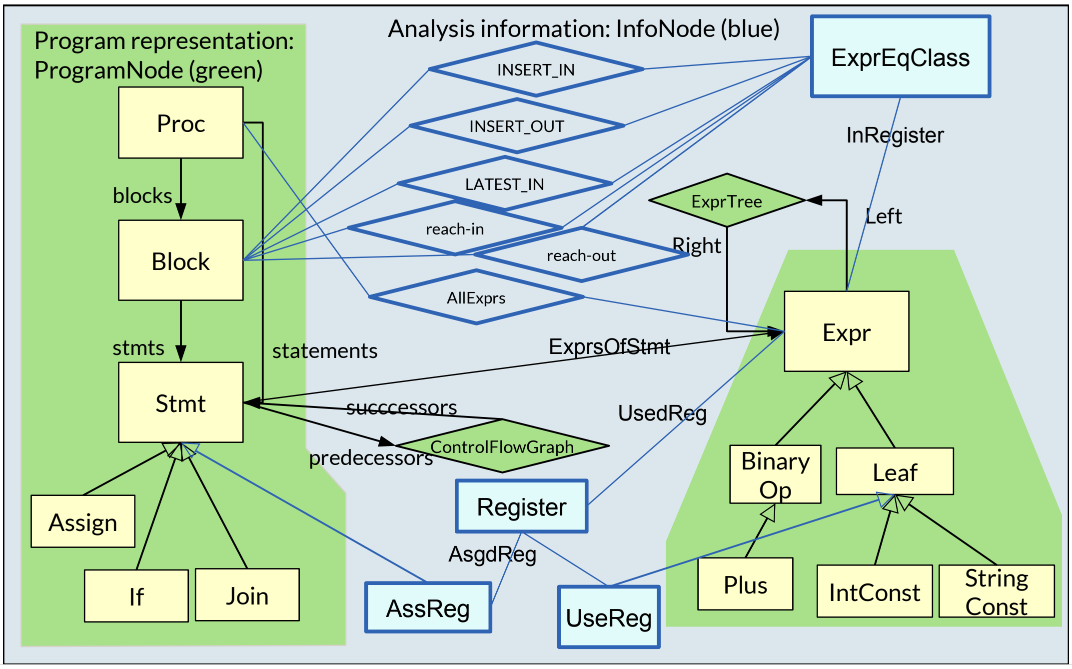
Model Analysis with Graph Reachability

- Use the **graph-logic-isomorphism:** Represent everything in a program or a model as directed graphs
  - Program code (control flow, statements, procedures, classes)
  - Model elements (states, transitions, ...)
  - Analysis information (abstract domains, flow info ...)
  - Directed graphs with node and edge types, node attributes, one-edge condition (no multi-graphs)
- **Use edge decomposition as textual notation**
- Use edge addition rewrite systems (EARS), Datalog and other graph reachability specification languages to
  - Query the graphs (on values and patterns)
  - Analyze the graphs (on reachability of nodes)
  - Map the graphs to each other (model mapping)
- Later: Use graph rewrite systems (GRS) to construct and augment the graphs, transform the graphs

Specification Process with Graph Rewrite Systems and EARS

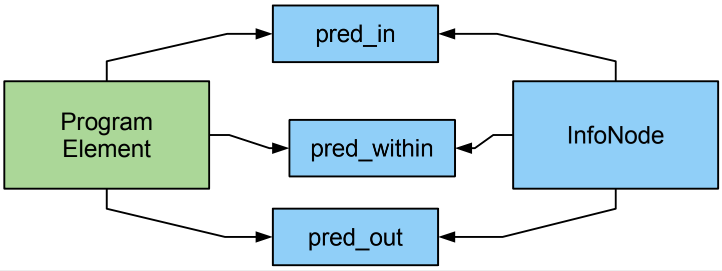
1) **Specification of the data model (graph schema)** with a graph-like DDL (ERD, MOF, GXL, UML or similar):
   - **Schema of the program representation**: program code as objects and basic relationships. This data, i.e., the start graph, is provided as result of the parser
   - **Schema of analysis information** (the infered predicates over the program objects) as objects or relationships
2) **Flat model and program analysis** (preparing the abstract interpretation)
   - Querying graphs, enlarging graphs, static slicing, Reachability
   - Equivalence classing
   - Materializing implicit knowledge to explicit knowledge
3) **Deep model and program analysis**
   - Inter-model reachability (traceability), materializing model mappings
   - **Abstract Interpretation** (program analysis as interpretation)
   - Specifying the transfer functions of an abstract interpretation of the program with graph rewrite rules on the analysis information
4) **Model and Program transformation** Transforming the program representation
   - Optimization such as peephole optimization or constant folding (context-free)
   - Code motion (Context-sensitive)

Q14: A Simple Program (Code) Model (Schema) in MOF



Deep Analysis and Abstract Interpretation with Graph Rewriting

- A ***graph-rewriting based abstract interpreter*** stores, for every program element of the program graph (Expr, Stmt, Block, Proc, Class < ProgramElement) three “truths” (values) for every node in the analysis information (InfoNode):
  - p:ProgramElement -:predicate_**in**-> i:InfoNode
    - // predicate_in(p,i)
  - p:ProgramElement -:predicate_**within**-> i:InfoNode
    - // predicate_within(p,i)
  - p:ProgramElement -:predicate_**out**-> i:InfoNode
    - // predicate_out(p,i)
- Values of program elements are encoded as an edge between program elements and InfoNodes



41.2. Reachability of Model Elements and Models for Model Analysis and Mapping

- With model mapping languages, such as edge addition rewrite systems or TGreQL

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

41.2.1. Simple Reachability of Model Elements and Models: Path Abbreviations in Graph Analysis

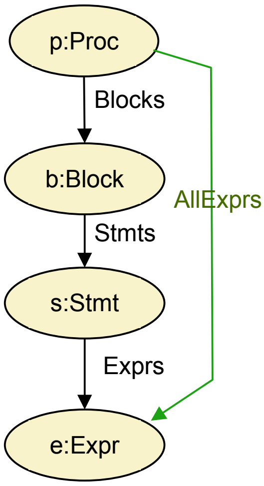
- With model mapping languages, such as edge addition rewrite systems or TGreQL

Path Abbreviations for Simple Reachability


```
-- GrGen notation:
rule collectAllExpr(p:Proc) {
  p -:Blocks-> b:Block;
  b -:Stmts-> s:Stmt;
  s -:Exprs-> e:Expr;
  modify {
     p -:AllExprs-> e;
  }
}
```

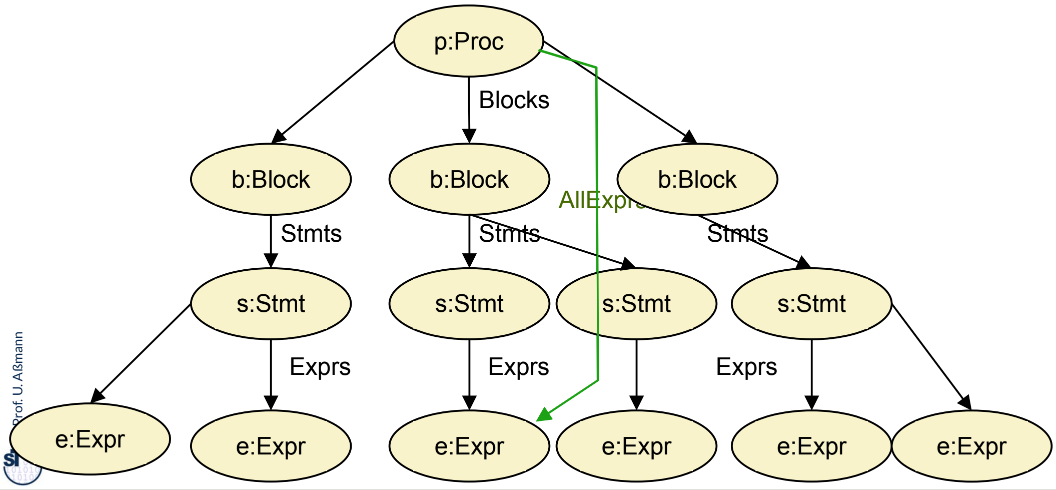
- Path abbreviations shorten paths in the manipulated graph.
- They may collect nodes into the neighborhood of other nodes.
- Ex.: Collection of Expressions for a procedure: edge addition

```
-- F-Datalog notation (edge decomposition):
AllExprs(Proc,Expr) :-
    Blocks(Proc,Block),
    Stmts(Block,Stmt),
    Exprs(Stmt,Expr).
-- if-then rules:
if  Blocks(Proc,Block),
    Stmts(Block,Stmt),
    Exprs(Stmt,Expr)
then
   AllExprs(Proc,Expr);
- regular expression notation (TGreQL):
AllExprs := Proc Blocks.Stmts.Exprs Expr
```



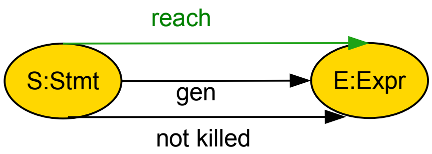
Forward Slicing from a Point in the ProgramGraph (Single-Source Multiple-Target (SSMT) Problems)

- A ***forward slice*** (SSMT-region) has one source, many targets and all intermediate nodes
- The slice border is the border of the region



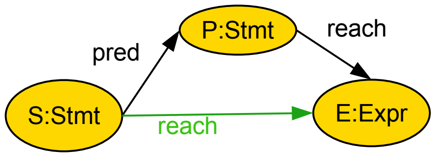
Linear Recursion for Remote Reachability

- Reachability most often can be reduced to transitive closure of one or several relations.
- Query: "Does an Stmt S reach a expression E?"
- TC combines path abbreviation with recursion
  - F-Datalog, GrGen: Left or right recursion
  - Kleene * in TGreQL
  - Thick arrow in Fujaba




```
// TGreQL
reach[gen]*(S:Stmt,E:Expr)

// GrGen can use inheritance on
// nodes and edges
rule reachability (s:Node) {
   s -:BasicEdge-> p:Node;
   p -:RecursiveEdge-> e:Node;
   modify {
   s -:RecursiveEdge-> e:Node
   }
}
```



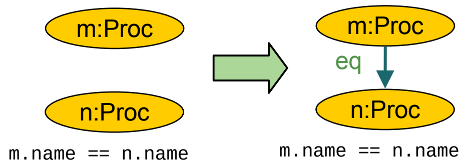

```
// F-Datalog
reach(S:Stmt,E:Expr) :- gen(S:Stmt,E:Expr), not killed(S:Stmt,E:Expr).
reach(S:Stmt,E:Expr) :- pred(S:Stmt,P), reach(P,E:Expr).
```

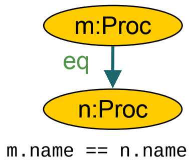
Ex.: Relating Nodes into Equivalence Classes

- Ex.: Computing equivalent nodes
- Context-sensitive problem, because m is not in the context of n

```
F-Datalog baserule:
eq(m:Proc,n:Proc) :-
   m.name == n.name.
-- If-then:
If (m:Proc, n:Proc) and m.name == n.name)
   eq(m,n)
}
- TgreQL regular expression:
m:Proc eq n.Proc if
m.name == n.name
```

```
// GrGen
rule buildGraph(m:Node, n:Node) {
    m.Name == n.Name;
    modify { m:-eq-> n }
}
```

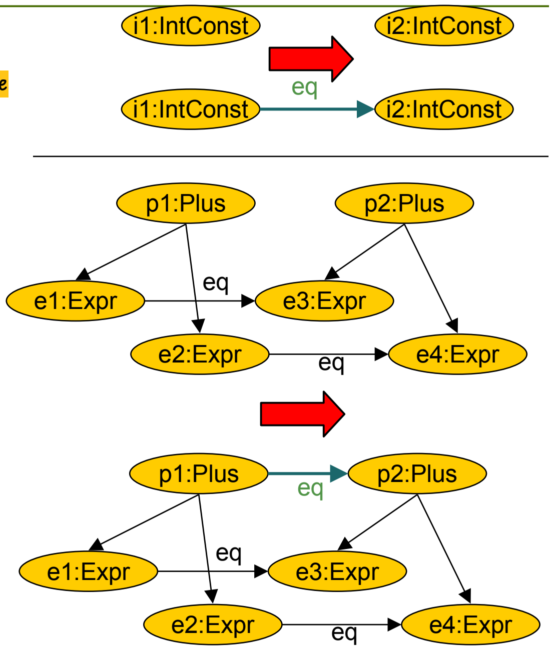




Ex. Relating Nodes into Equivalence Classes (Here: Value Numbering, Synt. Expression Equivalence)

- Ex.: Computing structurally equivalent expressions with bi-recursive reachability
- Question: *"Which expression trees have the same structure?"*

```
--- F-Datalog baserule:
eq(i1:IntConst,i2:IntConst) :-
   i1 ~= IntConst(Value),
   i2 ~= IntConst(Value).
--- recursive_rule:
eq(p1:Plus,p2:Plus) :-
   p1 ~= Plus(Type),
   p2 ~= Plus(Type),
   Left(p1,e1),
   Right(p1,e2),
   Left(p2,e3),
   Right(p2,e4).
   eq(e1,e3),
   eq(e2,e4).
```



41.3. Deep Model Analysis (Value-Flow Analysis, Data-Flow Analysis) as General (Recursive) Graph Reachability over Values

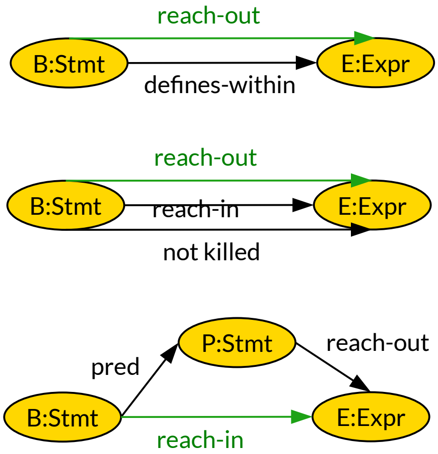
- with edge addition rewrite systems and F-Datalog

Data-flow Analysis for Reachability and Traceability

- **Value-flow analysis (data-flow analysis)** is a specific form of deep model analysis asking **reachability questions**, i.e., computing the *flow of data (value flow)* through the model or program, from variable assignments to variable uses
  - Result: the **value-flow graph (data-flow graph)**
  - If the value flow analysis is done along the control-flow graph, it is called an **abstract interpretation** of a program
    - EARS can do an abstract interpretation of a program, if they are rewriting on the control-flow graph. Then, their rules implement transfer functions of an abstract interpreter
- Examples of reachability problems:
  - **AllSuperClasses:** find out for a class transitively all superclasses
  - **AllEnclosingScopes:** find out for a scope all enclosing scopes
  - **AllEnclosingWholes**: find out, for a part, its wholes into which it is included
  - **Reaching Definitions Analysis:** Which Assignments (Definitions) of a variable can reach which statement?
  - **Live Variable Analysis:** At which statement is a variable live, will further be used?
  - **Busy Expression Analysis:** Which expression will be used on all outgoing paths?

Reaching Definition Analysis By Abstract Interpretation with EARS (Reachable Statements from Expression Definition)

- **Query:** "Which definitions of expressions reach which statement?"
  - Assignments of a variable, temporary, or register
  - Usually computed for all positions *before* and *after* a statement
- Graph rewrite rules implement an abstract interpreter
  - On instructions or on blocks of instructions
  - Flow information is expressed with edges of relations "reach-*"
- Recursive system (via edge reach-in)
  - (B reach-out E) := (E reaches end of block B)
- GrGen can express this via its generic reachability rules




```
reach-out(B,E) :- defines-within(B,E).
reach-out(B,E) :- reach-in(B,E), not killed(B,E).
reach-in(B,E)  :- pred(B,P), reach-out(P,E).
```

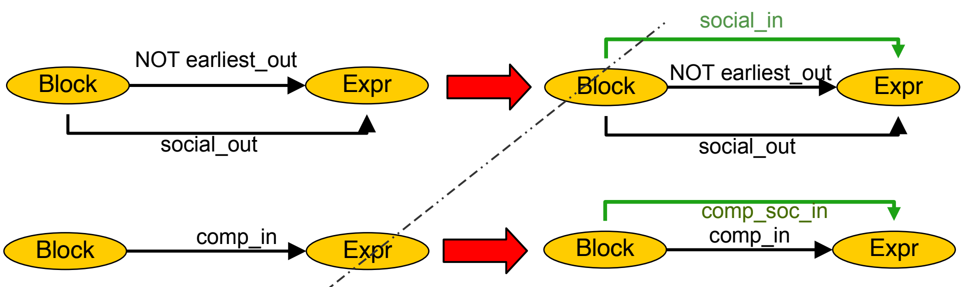
Code Motion Analysis

- **Code motion** is an essential transformation to speed up the generated code. However, it is a complex transformation:
  - Discovering loop-invariant expressions by data-flow analysis
  - Moving loop-invariant expressions out of loops upward
  - Code motion needs complex data-flow analysis
- **Busy Code Motion (BCM)** moves expressions as upward (early) as possible
- **Lazy Code Motion (LCM)**
  - Moving expressions out of loops to the front of the loop, upward, but carefully:
  - Moving expressions to an optimal place so that register lifetimes are shorter and not too long (optimally early)
  - LCM analysis computes this optimal early place of an expression [Knoop/Steffen]
    - Analyze an optimally early place for the placement of an expression
    - About 6 equation systems similar to reaching-definitions
  - Every equation system is an EARS [Aßmann00]

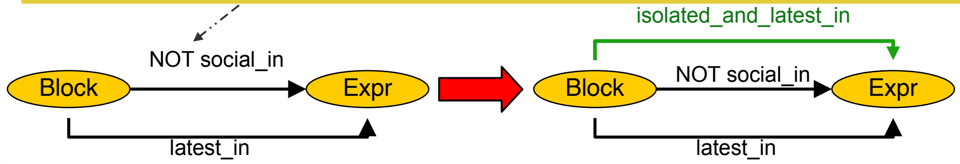
Excerpt from LCM Analysis with Overlaps

- Compute an optimally early block for an expression (out of a loop)

Query: "Which expression is not isolated (social) at the beginning of a block?"



Query: "Which expression is not isolated (social) at the beginning of a block?"

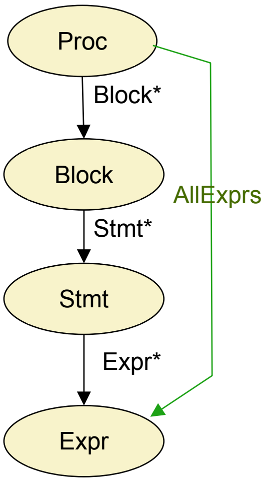


41.3.2 Regular Graph Reachability and Slicing

Regular Graph Reachability

- If the query can be expressed as a regular expression, the query is a **regular graph reachability problem**
- Kleene star is used as transitive closure operator
- TqreQL and Fujaba are languages offering Kleene *

```
-- F-Datalog notation:
AllExprs(Proc,Expr) :-
    Block*(Proc,Block),
    Stmt*(Block,Stmt),
    Expr*(Stmt,Expr).
-- GrGen if-then rules:
if  Proc -:Block*-> Block,
    Block -:Stmt*-> Stmt,
    Stmt -:Expr*-> Expr
modify {
   Proc -:AllExprs-> Expr
}
– regular expression notation (TGreQL):
AllExprs := Proc Block*.Stmt*.Expr* Expr
```



Static Slicing: Single-Source-Multiple-Target Regular Reachability (Regular Reachable Dependencies)

- [Weiser] [Tip]
- A **static slice** is the region of a program or model *dependent* from *one source* node (reachable by a regular reachability query in a dependency graph)
  - A static slice is a *single-source path regular reachability problem (SSPP)* on the dependency graph
  - A static slice introduces path abbreviations from one entity to a region
- A **forward slice** is a dependent region in *forward* direction of the program
  - The uses of a variable
  - The callees of a call
  - The uses of a type
- A **backward slice** is a dependent region in *backward* direction of the program
  - The assignments which can influence the value of a variable
  - The callers of a method
  - The type of a variable
- Slicing can map arbitrary entities in programs and models to other entities, based on a regular graph expression

Reachability within Models and Traceability between Models

- Data-flow analysis (graph reachability, slicing) can be done
  - Intraprocedurally (within one procedure)
  - Interprocedurally (program-wide)
- **Traceability** is inter-model slicing and graph reachability
  - inter-model: then it creates **trace relations** between requirements models, design models, and code models
  - Intra-megamodel: trace relations can trace dependencies between all models in a megamodel, e.g., in an MDA
- A **model mapping** is an inter-model trace(-ability) graph
  - Model mappings are very important for the dependency analysis and traceability in megamodels and the construction of macromodels

41.3.3 Context-Free Graph Reachability

- If arbitrary recursion patterns are allowed in F-Datalog and EARS queries, we arrive at context-free graph reachability.

Free Recursion

- Transitive closure and regular graph reachability rely on regular recursion (linear recursion) expressible with the Kleene-* on relations
- Beyond that, F-Datalog and EARS can describe other recursions
  - Context-free recursions
  - Cross-recursions
- Then, we speak of **context-free graph reachability**
  - A context-free language describes graph reachability
- Applications:
  - Complex intraprocedural value flow analyses
  - Interprocedural, whole-program analysis
  - Interprocedural IDFS framework (Reps)
  - Model mappings in a megamodel

41.4 More on the Logic-Graph Isomorphism

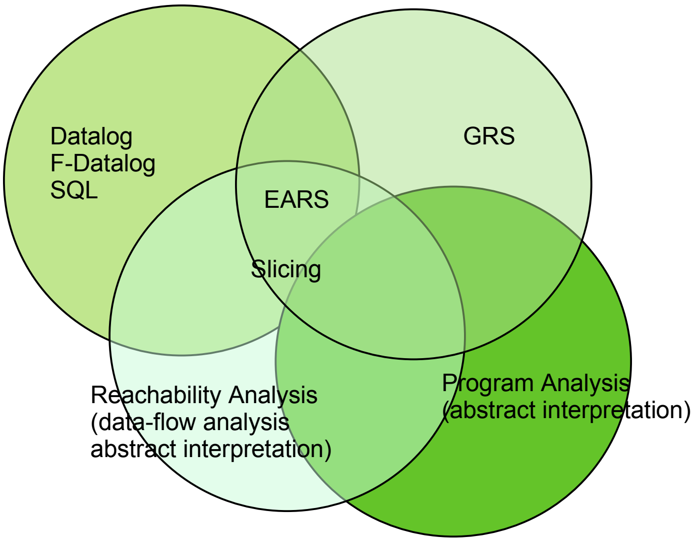
- [Courcelle] discovered that many problems can be expressed in logic (on facts) and in graph rewriting (on graphs)

Program and Model Analyses Covered by Graph Reachability

- Graph Reachability Analysis can do abstract interpretation
  - If it adds analysis information to the program elements and their control-flow graph
  - Slicing is a Single-Source-Multiple-Target reachability analysis
- Every abstract interpretation where a mapping of the abstract domains to graphs can be found.
  - Monotone and distributive data-flow analysis
    Control flow analysis (and callee analysis)
  - Static-single-assignment (SSA) construction
  - Interprocedural IDFS analysis framework (Reps)

The Common Core of Logic, Graph Rewriting and Program Analysis

- Graph rewriting, DATALOG and data-flow analysis have a common core: EARS
- Datalog query languages such as .QL or TgreQL can be extended by GRS



Relation DFA/F-DATALOG/GRS

- Abstract interpretation (Data-flow analysis), F-DATALOG and graph rewrite systems have a common kernel: EARS
  - As F-DATALOG, graph rewrite systems can be used to query the graph.
- Contrary to F-DATALOG and query languages, edge graph rewrite systems materialize their results instantly.
  - Therefore, they are amenable for *model analysis and mappings*
  - Graph rewriting is restricted to binary predicates and always yields all solutions
- General graph rewriting can do transformation, i.e. is much more powerful than F-DATALOG.
  - Graph rewriting enables a uniform view of the entire optimization process
  - There is no methodology on how to specify general abstract interpretations with graph rewrite systems
  - In interprocedural analysis, instead of chaotic iteration special evaluation strategies must be used [Reps95] [Knoop92]
  - Currently strategies have to be modeled in the rewrite specifications explicitly
- Uniform Specification of Analysis and Transformation [Aßmann00]
  - If the program analysis (including abstract interpretation) is specified with GRS, it can be unified with program transformation

41.4.1 Implementation of Data-Flow Analysis in Tools

Graph Rewrite Tools for Graph Reachability

- GrGen graph rewriting system (U Karlsruhe)
  - Www.grgen.net
- Fujaba graph rewrite system www.fujaba.de
- (e)MOFLON graph rewrite system www.moflon.de
  - TGG for Model Mapping, similar to QVT-R
  - See chapter MOFLON
- AGG graph rewrite system (From Berlin and Marburg)
  - http://user.cs.tu-berlin.de/~gragra/agg/
- VIATRA2 graph rewrite system on EMF
  - http://eclipse.org/gmt/VIATRA2/
- GROOVE for the construction of iInterpreters
  - http://groove.cs.utwente.nl/

Optimix: using Efficient Evaluation Algorithms from Logic Programming

- Tool OPTIMIX uses the „Order algorithm“ scheme [Aßmann00]
  - Generates target code of a programming language
    - Code generation uses variants of nested loop join algorithm
  - Works effectively on very sparse directed graphs
  - Bottom-up evaluation, as in F-Datalog; top-down evaluation as in Prolog possible, with resolution
- Optimizations from Datalog and F-Datalog
  - Bottom-up evaluation is normal, as in Datalog
  - Top-down evaluation as in Prolog possible, with resolution
  - Sometimes fixpoint evaluations can be avoided
  - Use of index structures possible
  - Linear bitvector union operations can be used
  - semi-naive evaluation
  - index structures
  - magic set transformation
  - transitive closure optimizations

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41.5 Model Mappings in In-Memory Megamodels (Modellverknüpfung) and Their Use for Traceability

- Model mapping languages are model query languages who enter their results again into the models as analysis information.
- They create *model mappings* which are important for macromodels.

Obligatory Literature

- [BERS08] Daniel Bildhauer, Jürgen Ebert, Volker Riediger, and Hannes Schwarz. Using the TGraph Approach for Model Fact Repositories. . In: Proceedings of the International Workshop on Model Reuse Strategies (MoRSe 2008). S. 9--18.
- Hannes Schwarz, Jürgen Ebert, and Andreas Winter. Graph-based traceability: a comprehensive approach. Software and System Modeling, 9 (4):473-492, 2010.

Inter-Model Analysis with Reachability

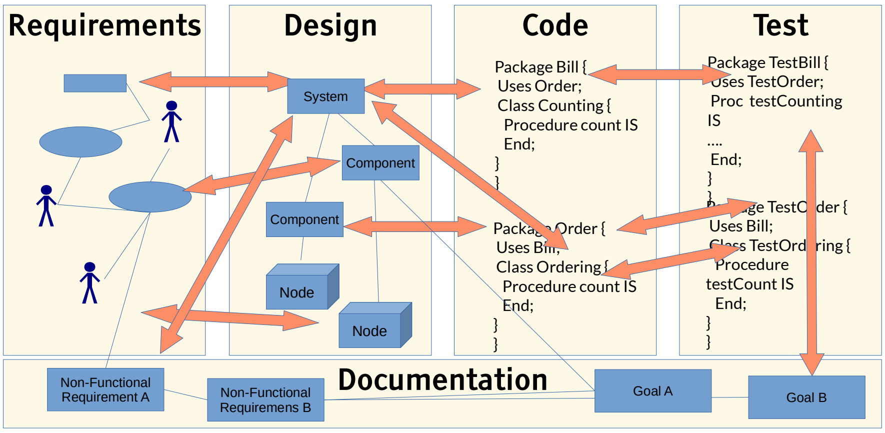
- **Deep model analysis:** Graph reachability analyzers create direct mappings (graphs) from indirect mappings (abbreviate intensional or recursive mappings)
  - for reachability of model elements
  - to create model slicings (projections to some subgraphs)
  - to prepare refactorings, transformers, and optimizers
    - For models: For model refactoring, adaptation and specialization, weaving and composition
    - For code: Portability to new processor types and memory hierarchies
  - For optimization (time, memory, energy consumption)
- For **traceability** of model elements in *other models.* Traceability is reachability of model elements over several models

Inter-Model Relationships in The ReDoDeCT Macromodel

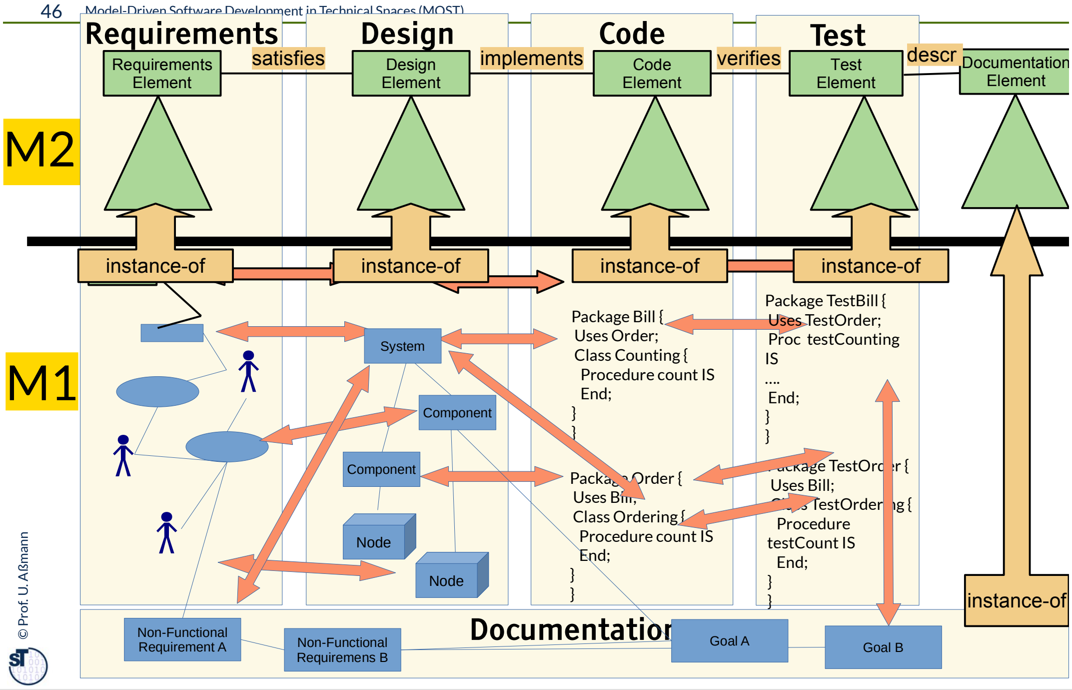
- An **inter-model relationship** is a relationship between model elements of different models
    - Here: expresses mapping between the Requirements model, Documentation, Design model, Code, Test cases
- The **ReDoDeCT macromodel relies on inter-model relationships** between all 4 models

Q12: The ReDoDeCT Problem and its Macromodel

- The **ReDoDeCT problem** is the problem how requirements, documentation, design, code, and tests are related (→ V model)
- Mappings between the Requirements model, Documentation files, Design model, Code, Test cases
- A **ReDoDeCT macromodel** has maintained mappings between all 5 models



- An (direct) inter-model relationship is defined between top-level metaclasses in the models of the macromodel
- The ReDoDeCT macromodel defines on direct inter-model relationships on RequirementsElement, DesignElement, CodeElement, TestElement, DocumentationElement



Specification of Traceability in ReDeCT with GrGen and TGreQL

[BERS08]

- **Direct inter-model relationships** form the basis of queries in the macromodel. Allow for the definition of
  - **Traceability relations** between model elements of different models
  - Hyperedges (tuples) between several model elements of different models
- **Any query language can be used for model mappings, if their results are entered into the model resp. macromodel**

```
// GrGen notation:
rule collectInterModelDep(r:Req, d:Des, c:Code, t:Test) {
  r -:reqs-> req:RequirementsElement;
     req.name="Count Bill";
  d -:arch-> archElem:DesignElement;
     archElem -:Satisfies->req;
  d -:design-> desElem:DesignElement;
     desElem -:Realize->archElem;
  c -:has-> class:Class;
    class -:Implements->desElem;
}
```

```
// Defining a inter-model hyperedge (tuple) in TGreQL [BERS08]
elementsIn(
  from req:V{RequirementsElement}, archElem:V{DesignElement},
   desElem:V{DesignElement}, class:V{ClassDefinition}
 with req.name="Count Bill"
    and req <-−{Satisfies} archElem
    and archElem <−−{Realize} desElem
    and desElem <−−{Implements} class
  report req, archElem, desElem, class
end
)
```

The End - Appendix Comprehension Questions

- Why do EARS correspond to binary Datalog? why is EARS a similar query language as .QL?
- Explain program slicing as an application of graph reachability.
- Why is regular graph reachability “regular”? What is the different to context-free graph reachability?
- How do you create a model mapping with regular graph reachability?
- Explain a typical data-flow analysis with EARS. Why do EARS rules that rewrite the information “around” the control-flow graph form an abstract interpreter?
- EARS can rewrite models. How would you specify a model refactoring engine with EARS?
- Why are EARS good for traceability in megamodels?